- 14 multiple choice questions worth 5 points each.
- 2 hand graded questions worth 15 points each.
- Exam covers sections 7.3, 7.4, 7.5, 7.8, 8.1, 8.2, 11.1, 11.2, and 11.3.

- No calculators!
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. Your ability to make your solution clear will be part of the grade.

Useful Formulas

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$
$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$	$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$
$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$	$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
$\sin(2\theta) = 2\sin\theta\cos\theta$	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$\int \csc x\, dx = -\ln\lvert\csc x + \cot x\rvert + C$	$\int \sec x\, dx = \ln\lvert\sec x + \tan x\rvert + C$
$\cosh t = \frac{1}{2}(e^t + e^{-t})$	$\sinh t = \frac{1}{2}(e^t - e^{-t})$
$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$
$\cosh^2 t = 1 + \sinh^2 t$	

1. Evaluate the integral $\displaystyle\int_1^2 \frac{1}{x^2\sqrt{x^2-1}}\,dx$.

   A. $\frac{1}{2}$

   B. $\frac{\sqrt{3}}{2}$

   C. $-\frac{1}{2}$

   D. $-\frac{\sqrt{3}}{2}$

   E. $\frac{\pi}{3}$

   F. $\cos 2 - \cos 1$

   G. $\sin 2 - \sin 1$

   H. The integral diverges

2. Which trig substitution would be helpful for evaluating the integral $\displaystyle\int \frac{1}{\sqrt{t^2-4t+13}}\,dt$?

A. $t = 2 + 3\sin\theta$

B. $t = 2 + 3\sec\theta$

C. $t = 2 + 3\tan\theta$

D. $t = 3 + 2\sin\theta$

E. $t = 3 + 2\sec\theta$

F. $t = 3 + 2\tan\theta$

G. $t = 3 - 2\sin\theta$

H. $t = 3 - 2\sec\theta$

I. $t = 3 - 2\tan\theta$

3. Integrate $\int_{-\pi}^{\frac{\pi}{2}} \cos(x)\cos(\sin(x))\,dx$.

   A. 2
   B. 3
   C. $\cos(1)$
   D. $\sin(5)$
   E. $\frac{\pi}{2}$
   F. $\sin(1)$
   G. π
   H. The integral diverges

4. Evaluate the improper integral $\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

   A. e^2
   B. $\frac{2}{e}$
   C. $\frac{-2}{e}$
   D. $2e$
   E. $-2e$
   F. $\frac{e}{2}$
   G. The integral diverges

5. Which of the following integrals diverge?

I. $\displaystyle\int_0^\infty \frac{dx}{2^x + 3^x}$ II. $\displaystyle\int_0^6 \frac{dx}{(6-x)^{1/3}}$ III. $\displaystyle\int_{-1}^1 \frac{dx}{x^2}$

A. I only
B. II only
C. III only
D. I and II
E. I and III
F. I, II, and III
G. None of the integrals diverge

6. Compute $\lim_{n\to\infty} (\ln(n+1) - \ln(n))$.

A. π
B. e
C. 1
D. 2
E. $\pi^2/6$
F. 0
G. The limit does not exist

7. Which integral represents the arc length of the curve $y = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{6}$?

A. $\int_0^{\frac{\pi}{6}} \cos x \, dx.$

B. $\int_0^{\frac{\pi}{6}} \sqrt{1 + \cos^2 x} \, dx.$

C. $\int_0^{\frac{\pi}{6}} \sec x \, dx.$

D. $\int_0^{\frac{\pi}{6}} \sqrt{1 + \sec^2 x} \, dx.$

E. $\int_0^{\frac{\pi}{6}} \ln x \, dx.$

F. $\int_0^{\frac{\pi}{6}} \sqrt{1 + \ln(x)^2} \, dx.$

G. $\int_0^{\frac{\pi}{6}} \ln(\cos x) \, dx.$

H. $\int_0^{\frac{\pi}{6}} \sqrt{1 + \ln(\cos x)^2} \, dx.$

I. $\int_0^{\frac{\pi}{6}} \ln(\sec x) \, dx.$

J. $\int_0^{\frac{\pi}{6}} \sqrt{1 + \ln(\sec x)^2} \, dx.$

8. Let $f(x) = \int_0^x \sqrt{t^2 + 6t + 8}\, dt$. Find the arc length of the curve $y = f(x)$, $0 \leq x \leq 4$

A. 15

B. 17

C. 19

D. 20

E. 22

F. 24

G. 25

9. Let C be the circle of radius 1 centered at $(3, 0)$. We get a donut if we revolve C around the y-axis. Which of the following integrals represents the surface area of the donut?

A. $\displaystyle\int_2^4 2\pi\sqrt{1-(x-3)^2}\,dx$

B. $\displaystyle\int_2^4 4\pi\sqrt{1-(x-3)^2}\,dx$

C. $\displaystyle\int_2^4 2\pi x\sqrt{1-(x-3)^2}\,dx$

D. $\displaystyle\int_2^4 4\pi x\sqrt{1-(x-3)^2}\,dx$

E. $\displaystyle\int_2^4 \frac{2\pi}{\sqrt{1-(x-3)^2}}\,dx$

F. $\displaystyle\int_2^4 \frac{4\pi}{\sqrt{1-(x-3)^2}}\,dx$

G. $\displaystyle\int_2^4 \frac{2\pi x}{\sqrt{1-(x-3)^2}}\,dx$

H. $\displaystyle\int_2^4 \frac{4\pi x}{\sqrt{1-(x-3)^2}}\,dx$

10. Beryl has a basil plant. Each morning, Beryl harvests ten leaves to make an amazing pesto sauce. Beryl keeps the plant under optimal temperature, lighting, and moisture conditions, so, over the course of the day, the number of basil leaves doubles. On Monday morning, prior to harvesting, Beryl's basil plant had 24 leaves. On Friday morning, Beryl decided to sell some of the basil because there was just too much of it.

    Compute the number of leaves of basil on Friday morning, prior to harvesting. Then decide which of the following numbers is closest to the answer.

    Hint: The situation could be described by a sequence a_n, where a_n is the number of basil leaves in the morning prior to harvesting on day n, where Monday is day 1. The problem tells us that $a_1 = 24$, and asks us for a_5. On day n, there are a_n leaves. After harvesting, there are $a_n - 10$ leaves. After doubling, there are $2(a_n - 10)$ leaves. That's exactly the number of leaves the next morning, that is, a_{n+1}.

    A. 30

    B. 40

    C. 50

    D. 60

    E. 70

    F. 80

    G. 90

    H. 100

    I. 110

    J. 120

    K. 130

11. Exactly one of the following sequences converges, which one is it?

A. $\{0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, \dots\}$

B. $a_n = 3^{-n}8^n$

C. $a_n = \dfrac{n}{(5+n^2)^{\frac{1}{3}}}$

D. $a_n = \dfrac{n}{\ln n}$

E. $a_n = (-1)^n e^n$

F. $a_n = \cos\left(\ln\left(\dfrac{n+1}{n}\right)\right)$

12. Find $\displaystyle\sum_{n=0}^{\infty} \frac{1+2^n}{5^n}$.

A. $\dfrac{2}{5}$

B. $\dfrac{5}{3}$

C. $\dfrac{5}{2}$

D. $\dfrac{35}{12}$

E. $\dfrac{10}{3}$

F. The series diverges

13. Evaluate the sum $\displaystyle\sum_{n=1}^{\infty} \frac{2}{(n+2)(n+3)}$.

Hint: $\frac{2}{(n+2)(n+3)} = \frac{2}{n+2} - \frac{2}{n+3}$.

A. $\frac{1}{6}$

B. 1

C. $\frac{2}{5}$

D. $\frac{7}{6}$

E. $\frac{2}{3}$

F. $\frac{1}{2}$

G. The series diverges

14. Exactly one of the following series diverges, which one is it?

A. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

B. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{n/2}}$

D. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

E. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

Written Problem. You will be graded on the readability and reasoning of your work.

15. Integrate $\displaystyle\int \frac{x^3 - 2x^2 - 7x + 6}{x^3 - 2x^2}\, dx$.

Written Problem. You will be graded on the readability and reasoning of your work.

16. Consider the triangle in the xy-plane with vertices $(1, 2)$, $(4, 2)$, and $(1, 6)$. Consider the shape formed by revolving this triangle about the x-axis. Compute the surface area of this shape.