- 14 multiple choice questions worth 5 points each.
- 2 hand graded questions worth 15 points each.
- Exam covers sections 7.3, 7.4, 7.5, 7.8, 8.1, 8.2, 11.1, 11.2, and 11.3.

- No calculators!
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. Your ability to make your solution clear will be part of the grade.

Useful Formulas

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$
$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$	$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$
$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$	$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
$\sin(2\theta) = 2\sin\theta\cos\theta$	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$\int \csc x\, dx = -\ln\lvert\csc x + \cot x\rvert + C$	$\int \sec x\, dx = \ln\lvert\sec x + \tan x\rvert + C$
$\cosh t = \frac{1}{2}(e^t + e^{-t})$	$\sinh t = \frac{1}{2}(e^t - e^{-t})$
$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$
$\cosh^2 t = 1 + \sinh^2 t$	

1. Evaluate the integral $\displaystyle\int_1^2 \frac{1}{x^2\sqrt{x^2-1}}\,dx$.

   A. $\frac{1}{2}$

   B. $\frac{\sqrt{3}}{2}$

   C. $-\frac{1}{2}$

   D. $-\frac{\sqrt{3}}{2}$

   E. $\frac{\pi}{3}$

   F. $\cos 2 - \cos 1$

   G. $\sin 2 - \sin 1$

   H. The integral diverges

Solution: Use the trigonometric substitution $x = \sec\theta$ so that $dx = \sec\theta\tan\theta\,d\theta$. You can find the new limits of integration by noting that if $x = 1$, then $1 = \sec\theta$ and $\theta = 0$ and that if $x = 2$, then $2 = \sec\theta$ and $\theta = \frac{\pi}{3}$. With these new values, we get

$$\begin{aligned}\int_0^{\pi/3} \frac{\sec\theta\tan\theta}{\sec^2\theta\sqrt{\sec^2\theta-1}}\,d\theta &= \int_0^{\pi/3} \frac{\tan\theta}{\sec\theta\sqrt{\tan^2\theta}}\,d\theta \\ &= \int_0^{\pi/3} \frac{1}{\sec\theta}\,d\theta \\ &= \int_0^{\pi/3} \cos\theta\,d\theta \\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

2. Which trig substitution would be helpful for evaluating the integral $\int \frac{1}{\sqrt{t^2-4t+13}}\,dt$?

A. $t = 2 + 3\sin\theta$

B. $t = 2 + 3\sec\theta$

C. $t = 2 + 3\tan\theta$

D. $t = 3 + 2\sin\theta$

E. $t = 3 + 2\sec\theta$

F. $t = 3 + 2\tan\theta$

G. $t = 3 - 2\sin\theta$

H. $t = 3 - 2\sec\theta$

I. $t = 3 - 2\tan\theta$

Solution: Complete the square:

$$t^2 - 4t + 13 = (t-2)^2 + 9$$

This is of the form $x^2 + a^2$ so we want to use $(t-2) = 3\tan\theta$ or $t = 2 + 3\tan\theta$.

3. Integrate $\int_{-\pi}^{\frac{\pi}{2}} \cos(x)\cos(\sin(x))\,dx$.

A. 2

B. 3

C. $\cos(1)$

D. $\sin(5)$

E. $\frac{\pi}{2}$

F. $\sin(1)$

G. π

H. The integral diverges

Solution: Use Substitution, letting $u = \sin x$ and $du = \cos x\ dx$

$$\begin{aligned}\int_{-\pi}^{\pi/2} \cos(x)\cos(\sin(x))\,dx &= \int_{x=-\pi}^{x=\pi/2} \cos u\ du \\ &= \sin u\Big|_{x=-\pi}^{x=\pi/2} = \sin(\sin x)\Big|_{x=-\pi}^{x=\pi/2} \\ &= \sin(\sin(\pi/2)) - \sin(\sin(-\pi)) \\ &= \sin(1) - \sin(0) = \sin(1)\end{aligned}$$

4. Evaluate the improper integral $\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

A. e^2

B. $\frac{2}{e}$

C. $\dfrac{-2}{e}$

D. $2e$

E. $-2e$

F. $\dfrac{e}{2}$

G. The integral diverges

Solution: Let $u = -\sqrt{x}$ so $du = -\frac{1}{2\sqrt{x}}\,dx$

$$\begin{aligned}\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx &= \lim_{t\to\infty}\int_1^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx \\ &= \lim_{t\to\infty}\int_{x=1}^{x=t} e^u(-2)\,du \\ &= \lim_{t\to\infty} -2e^u\Big|_{x=1}^{x=t} = \lim_{t\to\infty} -2e^{-\sqrt{x}}\Big|_{x=1}^{x=t} = \lim_{t\to\infty} -2e^{-\sqrt{t}} + 2e^{-1} = 0 + 2e^{-1}\end{aligned}$$

5. Which of the following integrals diverge?

I. $\displaystyle\int_0^\infty \frac{dx}{2^x+3^x}$ II. $\displaystyle\int_0^6 \frac{dx}{(6-x)^{1/3}}$ III. $\displaystyle\int_{-1}^1 \frac{dx}{x^2}$

A. I only

B. II only

C. III only

D. I and II

E. I and III

F. I, II, and III

G. None of the integrals diverge

Solution:

I. For this we make a comparison.

$$\int_0^\infty \frac{dx}{2^x+3^x} \le \int_0^\infty \frac{dx}{e^x}$$

and $\int_0^\infty e^{-x}\,dx$ converges, so the integral in question converges too.

II. For this integral we do a substitution and note that we have a $\frac{1}{x^p}$ integral. Let $u = 6-x$:

$$\int_0^6 \frac{dx}{(6-x)^{1/3}} = \int_6^0 -\frac{du}{u^{1/3}}$$

and this integral (with vertical asymptote at $u=0$) converges because $p = 1/3 < 1$.

III. This integral has vertical asymptote at $x = 0$, thus

$$\int_{-1}^1 \frac{dx}{x^2} = \int_{-1}^0 \frac{dx}{x^2} + \int_0^1 \frac{dx}{x^2}$$

and, each of these diverge because $\int_0^1 \frac{1}{x^p}\,dx$ diverges when $p \ge 1$.

6. Compute $\lim_{n\to\infty} (\ln(n+1) - \ln(n))$.

A. π

B. e

C. 1

D. 2

E. $\pi^2/6$

F. 0

G. The limit does not exist

Solution:

$$\lim_{n\to\infty} (\ln(n+1) - \ln(n)) = \lim_{n\to\infty} \ln\left(\frac{n+1}{n}\right)$$
$$= \ln\left(\lim_{n\to\infty} \frac{n+1}{n}\right) = \ln(1) = 0$$

7. Which integral represents the arc length of the curve $y = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{6}$?

A. $\displaystyle\int_0^{\frac{\pi}{6}} \cos x \, dx.$

B. $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{1 + \cos^2 x} \, dx.$

C. $\displaystyle\int_0^{\frac{\pi}{6}} \sec x \, dx.$

D. $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{1 + \sec^2 x} \, dx.$

E. $\displaystyle\int_0^{\frac{\pi}{6}} \ln x \, dx.$

F. $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{1 + \ln(x)^2} \, dx.$

G. $\displaystyle\int_0^{\frac{\pi}{6}} \ln(\cos x) \, dx.$

H. $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{1 + \ln(\cos x)^2} \, dx.$

I. $\displaystyle\int_0^{\frac{\pi}{6}} \ln(\sec x) \, dx.$

J. $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{1 + \ln(\sec x)^2} \, dx.$

Solution:

$$L = \int_0^{\frac{\pi}{6}} \sqrt{1 + (y')^2} \, dx = \int_0^{\frac{\pi}{6}} \sqrt{1 + [(\ln(\cos x))']^2} \, dx = \int_0^{\frac{\pi}{6}} \sqrt{1 + \tan^2 x} \, dx$$
$$= \int_0^{\frac{\pi}{6}} \sec x \, dx$$

8. Let $f(x) = \int_0^x \sqrt{t^2 + 6t + 8}\, dt$. Find the arc length of the curve $y = f(x)$, $0 \le x \le 4$

A. 15

B. 17

C. 19

D. 20

E. 22

F. 24

G. 25

Solution: Here you need to note that $f'(x) = \sqrt{x^2 + 6x + 8}$ and thus $(f'(x))^2 = x^2 + 6x + 8$.

$$L = \int_0^4 \sqrt{1 + (f'(x))^2}\, dx = \int_0^4 \sqrt{1 + (x^2 + 6x + 8)}\, dx = \int_0^4 \sqrt{x^2 + 6x + 9}\, dx$$

$$= \int_0^4 \sqrt{(x+3)^2}\, dx = \int_0^4 x + 3\, dx = \frac{x^2}{2} + 3x \Big|_0^4 = 8 + 12 = 20$$

9. Let C be the circle of radius 1 centered at $(3, 0)$. We get a donut if we revolve C around the y-axis. Which of the following integrals represents the surface area of the donut?

A. $\displaystyle\int_2^4 2\pi\sqrt{1-(x-3)^2}\,dx$

B. $\displaystyle\int_2^4 4\pi\sqrt{1-(x-3)^2}\,dx$

C. $\displaystyle\int_2^4 2\pi x\sqrt{1-(x-3)^2}\,dx$

D. $\displaystyle\int_2^4 4\pi x\sqrt{1-(x-3)^2}\,dx$

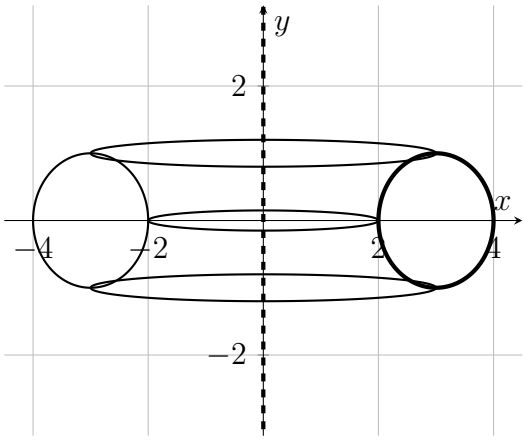
E. $\displaystyle\int_2^4 \frac{2\pi}{\sqrt{1-(x-3)^2}}\,dx$

F. $\displaystyle\int_2^4 \frac{4\pi}{\sqrt{1-(x-3)^2}}\,dx$

G. $\displaystyle\int_2^4 \frac{2\pi x}{\sqrt{1-(x-3)^2}}\,dx$

H. $\displaystyle\int_2^4 \frac{4\pi x}{\sqrt{1-(x-3)^2}}\,dx$

Solution: The equation of the circle C is $(x-3)^2+y^2=1$, or equivalently $y=\pm\sqrt{1-(x-3)^2}$, where x goes from 2 to 4.



Consider the upper half plane where $y \geq 0$, the surface area of the donut in the upper half plane is exactly half of the surface area of the whole donut (thus the 2 in front of the integral).

$$
\begin{aligned}
S &= 2\int 2\pi r\, ds \\
&= 2\int_2^4 2\pi x\sqrt{1+(y')^2}\, dx = 2\int_2^4 2\pi x\sqrt{1+((\sqrt{1-(x-3)^2})')^2}\, dx \\
&= \int_2^4 4\pi x\sqrt{1+\frac{(x-3)^2}{1-(x-3)^2}}\, dx = \int_2^4 \frac{4\pi x}{\sqrt{1-(x-3)^2}}\, dx
\end{aligned}
$$

10. Beryl has a basil plant. Each morning, Beryl harvests ten leaves to make an amazing pesto sauce. Beryl keeps the plant under optimal temperature, lighting, and moisture conditions, so, over the course of the day, the number of basil leaves doubles. On Monday morning, prior to harvesting, Beryl's basil plant had 24 leaves. On Friday morning, Beryl decided to sell some of the basil because there was just too much of it.

    Compute the number of leaves of basil on Friday morning, prior to harvesting. Then decide which of the following numbers is closest to the answer.

    Hint: The situation could be described by a sequence a_n, where a_n is the number of basil leaves in the morning prior to harvesting on day n, where Monday is day 1. The problem tells us that $a_1 = 24$, and asks us for a_5. On day n, there are a_n leaves. After harvesting, there are $a_n - 10$ leaves. After doubling, there are $2(a_n - 10)$ leaves. That's exactly the number of leaves the next morning, that is, a_{n+1}.

    A. 30
    B. 40
    C. 50
    D. 60
    E. 70
    F. 80
    G. 90
    H. 100
    I. 110
    J. 120
    K. 130

Solution: The hint gives us that $a_{n+1} = 2(a_n - 10)$. We compute that

$$
\begin{aligned}
a_1 &= 24, \\
a_2 &= 2(24 - 10) = 28, \\
a_3 &= 2(28 - 10) = 36, \\
a_4 &= 2(36 - 10) = 52, \\
a_5 &= 2(52 - 10) = \boxed{84}.
\end{aligned}
$$

The answer choice closest to 84 is F.

11. Exactly one of the following sequences converges, which one is it?

A. $\{0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, \dots\}$

B. $a_n = 3^{-n}8^n$

C. $a_n = \dfrac{n}{(5+n^2)^{\frac{1}{3}}}$

D. $a_n = \dfrac{n}{\ln n}$

E. $a_n = (-1)^n e^n$

F. $a_n = \cos\left(\ln\left(\dfrac{n+1}{n}\right)\right)$

Solution:

A. This does not converge since the numbers do not eventually approach a single value.

B. $\lim\limits_{n\to\infty} \dfrac{8^n}{3^n}$ diverges to ∞.

C. $\lim\limits_{n\to\infty} \left(\dfrac{n^3}{5+n^2}\right)^{1/3} = \left(\lim\limits_{n\to\infty} \dfrac{n^3}{5+n^2}\right)^{1/3}$ and L'Hopital can be applied to see that it diverges to ∞.

D. L'Hopital gives $\lim\limits_{n\to\infty} \dfrac{n}{\ln n} = \lim n$ which diverges to ∞.

E. clearly diverges (write down a few terms).

F. Since $\lim \frac{n+1}{n} = 1$ this sequence has limit $\cos(\ln 1) = \cos 0 = 1$.

12. Find $\displaystyle\sum_{n=0}^{\infty} \frac{1+2^n}{5^n}$.

A. $\frac{2}{5}$

B. $\frac{5}{3}$

C. $\frac{5}{2}$

D. $\frac{35}{12}$

E. $\frac{10}{3}$

F. The series diverges

Solution: We can simplify the sequence to be summed as

$$\frac{1+2^n}{5^n} = \frac{1}{5^n} + \frac{2^n}{5^n} = \left(\frac{1}{5}\right)^n + \left(\frac{2}{5}\right)^n.$$

Using our rules for working with series and the formula for a geometric series, we find that

$$\sum_{n=0}^{\infty} \left(\left(\tfrac{1}{5}\right)^n + \left(\tfrac{2}{5}\right)^n\right) = \sum_{n=0}^{\infty} \left(\tfrac{1}{5}\right)^n + \sum_{n=0}^{\infty} \left(\tfrac{2}{5}\right)^n = \frac{1}{1-\frac{1}{5}} + \frac{1}{1-\frac{2}{5}} = \frac{5}{4} + \frac{5}{3} = \frac{35}{12}.$$

13. Evaluate the sum $\displaystyle\sum_{n=1}^{\infty} \frac{2}{(n+2)(n+3)}$.

Hint: $\frac{2}{(n+2)(n+3)} = \frac{2}{n+2} - \frac{2}{n+3}$.

A. $\frac{1}{6}$

B. 1

C. $\frac{2}{5}$

D. $\frac{7}{6}$

E. $\frac{2}{3}$

F. $\frac{1}{2}$

G. The series diverges

Solution: Thus, the Nth partial sum is

$$s_N = \sum_{n=1}^{N} \left(\frac{2}{n+2} - \frac{2}{n+3} \right) = \left(\frac{2}{3} - \frac{2}{4} \right) + \left(\frac{2}{4} - \frac{2}{5} \right) + \left(\frac{2}{5} - \frac{2}{6} \right) + \cdots + \left(\frac{2}{N+2} - \frac{2}{N+3} \right)$$
$$= \frac{2}{3} - \frac{2}{N+3}$$

The limit of the partial sums is $\frac{2}{3}$.

14. Exactly one of the following series diverges, which one is it?

A. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

B. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{n/2}}$

D. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

E. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

Solution: A is an integral test and $\int_1^{\infty} \frac{1}{x^2}\,dx$ converges so therefore the series converges.

B is comparable, by integral test to $\int_2^{\infty} \frac{1}{x \ln x}\,dx = \ln(\ln x)$, which diverges.

C is a geometric series with $r = 2^{-1/2} = 1/\sqrt{2} < 1$, which converges.

D is a p series with $p > 1$ so therefore converges.

E: converges by an integral test or you could compare as $a_n \leq \frac{\sqrt{n}}{n^3}$, which converges.

Written Problem. You will be graded on the readability and reasoning of your work.

15. Integrate $\displaystyle\int \frac{x^3-2x^2-7x+6}{x^3-2x^2}\,dx$.

Solution: We are tasked with integrating a rational function, so we use the method of partial fractions. The first step is to do long division. In fact, in this case we could just directly split $\frac{x^3-2x^2-7x+6}{x^3-2x^2}$ into $\frac{x^3-2x^2}{x^3-2x^2}+\frac{-7x+6}{x^3-2x^2}$. Or, just do the long division

$$\begin{array}{r} 1 \\ x^3-2x^2\overline{)\;x^3-2x^2-7x+6} \\ \underline{-x^3+2x^2} \end{array}$$

In any case, we find that

$$\frac{x^3-2x^2-7x+6}{x^3-2x^2} = 1+\frac{-7x+6}{x^3-2x^2}$$

Next, we factor $x^3-2x^2 = x^2(x-2)$. The method of partial fractions tells us that to deal with the remainder term, we can split it up in the form

$$\frac{-7x+6}{x^3-2x^2} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-2}.$$

Clearing the denominator, we find that

$$-7x+6 = Ax(x-2)+B(x-2)+Cx^2.$$

You can take this and find equations for A, B and C or you can pick clever values to plug in for x. The equations you might find are
$A+C=0$
$-2A+B=-7$
$-2B=6$

If you want to plug values in for x to get many terms to be zero, we should plug in $x=0$ and $x=2$. Plugging in $x=0$, we find that $6=B(-2)$, so $B=-3$. Plugging in $x=2$, we find that $-14+6=C(4)$, so $C=-2$. To find A, we should plug something else in, and $x=1$ is an easy choice. We find that

$$\begin{aligned} -7+6 &= A(1)(-1)+B(-1)+C(1)^2 \\ -1 &= -A-B+C = -A+3-2. \end{aligned}$$

Thus, $A = 2$. We conclude that

$$\frac{x^3 - 2x^2 - 7x + 6}{x^3 - 2x^2} = 1 + \frac{2}{x} - \frac{3}{x^2} - \frac{2}{x-2}.$$

Integrating, we find that

$$\int \frac{x^3 - 2x^2 - 7x + 6}{x^3 - 2x^2}\, dx = \boxed{x + 2\ln|x| + \frac{3}{x} - 2\ln|x-2|\,.} + C$$

Written Problem. You will be graded on the readability and reasoning of your work.

16. Consider the triangle in the xy-plane with vertices $(1,2)$, $(4,2)$, and $(1,6)$. Consider the shape formed by revolving this triangle about the x-axis. Compute the surface area of this shape.

Solution: The triangle has three sides. Each side contributes to the surface area. Let's start with the side from $(4,2)$ to $(1,6)$. Since the side is straight, the surface of revolution is part of a cone. In class, we saw that the area of a part of a cone is 2π(average radius)(slant length). Since we are revolving about the x-axis, the radius is y. The y-value ranges from 2 to 6, so its average value is 4. Using the distance formula, the slant length is $\sqrt{3^2+4^2}=5$. Thus, the area of this part of the shape is $2\pi(4)(5)=40\pi$.

Next, we consider the side from $(1,2)$ to $(4,2)$. This is a horizontal line, so when we revolve it about the x-axis, we get a cylinder. The radius of the cylinder is y, which is 2 for this side. The length of the cylinder is $4-1=3$. Thus, the area of the cylinder is $2\pi(2)(3)=12\pi$.

Finally, we consider the side from $(1,2)$ to $(1,6)$. When we revolve it around the x-axis, we obtain an annulus, that is, a washer. The inner radius of the annulus is $y=2$, and the outer radius of the annulus is $y=6$. Thus, the area of the washer is $\pi(6^2-2^2)=32\pi$.

The total surface area is $40\pi+12\pi+32\pi=\boxed{84\pi}$.

We can also find that the equation of the line joining $(4,2)$ and $(1,6)$ is $4x+3y=22$. Thus, $y=\frac{1}{3}(22-4x)$. Thus, the area of this part of the shape is

$$\begin{aligned}
\int_{x=1}^{4} 2\pi y\,ds &= 2\pi\int_1^4 \tfrac{1}{3}(22-4x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \\
&= 2\pi\int_1^4 \tfrac{1}{3}(22-4x)\sqrt{1+\left(-\tfrac{4}{3}\right)^2}\,dx \\
&= 2\pi\int_1^4 \tfrac{1}{3}(22-4x)\sqrt{\tfrac{25}{9}}\,dx \\
&= 2\pi\int_1^4 \tfrac{1}{3}(22-4x)\cdot\tfrac{5}{3}\,dx \\
&= \frac{10\pi}{9}\left(22x-2x^2\right)\Big|_{x=1}^{4} \\
&= \frac{10\pi}{9}\left(22\cdot(4-1)-2\cdot(16-1)\right) \\
&= \frac{10\pi}{9}(66-30) \\
&= 40\pi.
\end{aligned}$$

Equivalently, we can compute in terms of y that $x = \frac{1}{4}(22 - 3y)$, so the area is

$$\begin{aligned}
\int_{y=2}^{6} 2\pi y \, ds &= 2\pi \int_2^6 y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \\
&= 2\pi \int_2^6 y\sqrt{1 + \left(-\tfrac{3}{4}\right)^2}\, dy \\
&= 2\pi \int_2^6 y\sqrt{\tfrac{25}{16}}\, dy \\
&= 2\pi \int_2^6 y \cdot \tfrac{5}{4}\, dy \\
&= \frac{5\pi}{2}\left(\tfrac{1}{2}y^2\right)\Big|_{y=2}^{6} \\
&= \frac{5\pi}{2}\tfrac{1}{2}(6^2 - 2^2) \\
&= \frac{5\pi}{2} \cdot 16 \\
&= 40\pi.
\end{aligned}$$

Likewise, we could find that the equation of the line joining $(1, 2)$ and $(4, 2)$ is $y = 2$. Then, the surface area is

$$\begin{aligned}
\int_{x=1}^{4} 2\pi y \, ds &= 2\pi \int_1^4 y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \\
&= 2\pi \int_1^4 2\sqrt{1 + 0^2}\, dx \\
&= 2\pi \int_1^4 2\, dx \\
&= 2\pi(4 - 1)(2) \\
&= 12\pi.
\end{aligned}$$

There is no way to do this part of the problem in terms of y, since we cannot solve for x in terms of y.

Finally, we can find that the equation of the line joining $(1, 2)$ and $(1, 6)$ is $x = 1$. Then, the surface area is

$$\begin{aligned}
\int_{y=2}^{6} 2\pi y \, ds &= 2\pi \int_{2}^{6} y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \\
&= 2\pi \int_{2}^{6} y\sqrt{1 + 0^2}\, dx \\
&= 2\pi \int_{2}^{6} y \, dx \\
&= 2\pi \left(\tfrac{1}{2}y^2\right)\Big|_{y=2}^{6} \\
&= 2\pi \cdot \tfrac{1}{2}(6^2 - 2^2) \\
&= 32\pi.
\end{aligned}$$

There is no way to do this part of the problem in terms of x, because we cannot solve for y in terms of x.