Math 132 - December 1, 2017

Section 11.9: Functions as Power Series

- Manupulating known power series (geometric): Substitution, Addition, Multiplication, Division (difficult), Differentiation, Integration

1. **Clicker** (Method: multiplication)

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$$
$$\frac{1}{2+x} = \frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} + \frac{x^4}{32} - \cdots$$

Use multiplication of power series to find a series for $\frac{1}{2+x} \cdot \ln(1+x)$.
(This is definitely a pain, but just distribute the terms out.)

$$\frac{1}{2+x} \cdot \ln(1+x) = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots\right)\left(\frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} + \frac{x^4}{32} - \cdots\right)$$

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{n+1}} x^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^{n+1}} x^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{n+1}} x^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{2^{n+1}} x^n$

(e) Something else

2. (Method: Long Division–Challenging and worth avoiding whenever possible)
Use long division of power series to find a series for $\frac{\ln(1+x)}{\ln(1-x)}$
Note: $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \cdots$
and: $\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 - \cdots$

3. (Method: Integration. Note $\int \frac{1}{1+x^2}\, dx = \arctan x + C$)
Find a series for $\arctan x$.

4. Use your previous series to find a series for $\arctan(1) = \pi/4$

5. Find a series for $\frac{1}{1+x^{10}}$ and use your series to approximate $\displaystyle\int_0^{0.25} \frac{1}{1+x^{10}}\, dx$

Section 11.10: Taylor Series

- Goal: Given a function: find a power series that equal the function
- Taylor Series
  $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ where $c_n = \dfrac{f^{(n)}(a)}{n!}$.
- Maclaurin Series (Taylor Series with $a = 0$)
  $f(x) = \sum_{n=0}^{\infty} c_n x^n$ where $c_n = \dfrac{f^{(n)}(0)}{n!}$.

6. Let $f(x) = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$
   (a) Find $f(0)$
   (b) Find $f(1)$
   (c) Find $f'(x) =$
   (d) Find $f''(x) =$
   (e) Graph $f(x)$.
7. Find the Taylor Series for $f(x) = \sin x$ centered at $x = 0$
   (Make a table for the derivatives!)
8. Find the Taylor Series for $f(x) = \cos x$ centered at $x = 0$
9. Find the Taylor Series for $f(x) = 4 + 2x - 3x^2 - x^3 + 7x^4 - x^5$ centered at $x = 0$.
10. Find the Taylor Series for $f(x) = 4 + 2x - 3x^2 - x^3 + 7x^4 - x^5$ centered at $x = 1$.