Math 132 - November 29, 2017

Section 11.8: Power Series

- Center of the Series, Radius of Convergence, Interval of Convergence

Section 11.9: Functions as Power Series

- Manupulating known power series (geometric): Substitution, Addition, Multiplication, Division (difficult), Differentiation, Integration

1. **Clicker** Given the sum of the geometric series below:

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + \cdots = \frac{1}{1-x}$$

What is the sum:

$$1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 + \cdots$$

(a) $\frac{1}{1-x}$ (b) $\frac{1}{1+x}$ (c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1+x^2}$

2. (Method: Substitution)
Find a series for $\frac{1}{1+x^2}$

3. **Clicker** Find a power series for $f(x) = \dfrac{x^3}{4+x}$

(a) $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^n x^n$

(b) $\displaystyle\sum_{n=0}^{\infty} 4^n x^n$

(c) $\displaystyle\sum_{n=0}^{\infty} (-4)^{n+1} x^{n+3}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}} x^{n+3}$

(e) There is no series for $f(x)$.

4. Reindex the solution from the previous question so that the power of x is x^n.

5. (Method: Differentiation)
   Find a series for $\frac{1}{(1-x)^3}$

6. (Method: Integration. Note $\int \frac{1}{1+x^2}\,dx = \arctan x + C$)
   Find a series for $\arctan x$.

7. Use your previous series to find a series for $\arctan(1) = \pi/4$

8. Find a series for $\frac{1}{1+x^{10}}$ and use your series to approximate $\displaystyle\int_0^{0.25} \frac{1}{1+x^{10}}\,dx$

9. (Method: multiplication)
   Use multiplication of power series to find a series for $\frac{1}{1-x} \cdot \frac{1}{1-x}$.
   (This is definitely a pain, but just distribute the terms out.)

   $$(1 + x + x^2 + x^3 + x^4 + \cdots)(1 + x + x^2 + x^3 + x^4 + \cdots) =$$

10. (Method: Long Division–Challenging and worth avoiding whenever possible)
    Use long division of power series to find a series for $\frac{\ln(1+x)}{\ln(1-x)}$
    Note: $\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \cdots$
    and: $\ln(1 - x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 - \cdots$

11. Find power series for the following functions (you figure out the method(s) to use!)

    (a) $\dfrac{1}{4+3x}$

    (b) $\dfrac{x}{(1+x)^2}$

    (c) $\dfrac{1+2x}{1-x} =$