Math 132 - November 17, 2017

Section 11.5: Alternating Series

Let $S = \sum(-1)^n b_n$ where $b_n > 0$ be an alternating series. S converges if the following are true:

- $b_{n+1} < b_n$ (so the b_n are decreasing)
- $\lim b_n = 0$

And $|R_n| = |S - S_n| \leq b_{n+1}$

Section 11.6: Absolute Convergence

- Definition: $\sum a_n$ is **absolutely convergent** if $\sum |a_n|$ converges.
- Definition: $\sum a_n$ is **conditionally convergent** if $\sum a_n$ converges but $\sum |a_n|$ diverges.
- **Theorem:** If $\sum a_n$ is absolutely convergent then $\sum a_n$ is convergent.

1. **Clicker** **Alternating Series:** Does the series converge or diverge:
   $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \cdots$$
   (a) Converge (b) Diverge

2. **Clicker** Converge or Diverge? $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$
   If it converges, find the sum accurate to 0.01.
   (a) Converge (b) Diverge

3. Converge or Diverge? $\frac{1}{4} - \frac{2}{5} + \frac{4}{6} - \frac{5}{7} + \cdots$
   If it converges, find the sum accurate to 0.01.

4. Converge or Diverge? $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \cdots$
   If it converges, find the sum accurate to 0.1.

5. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$.
   (a) $S_{15} = 0.8245417573\ldots$.
   How close does the alternating series remainder theorem guarantee S_{15} to be from S?
   (b) If you want the partial sum to be within 0.0001 of the sum, how many terms do you need to take?

6. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$.
   (a) $S_{15} = 0.901676118\ldots$.
   How close does the alternating series remainder theorem guarantee S_{15} to be from S?
   (b) If you want the partial sum to be within 0.001 of the sum, how many terms do you need to take?

7. **Clicker** Does the series converge absolutely: $\sum_{i=1}^{\infty} \frac{(-1)^{n+1}}{n^2+2n+1}$

   (a) Yes (b) No (c) Converges conditionally (d) Diverges (e) Diverges conditionally

8. Are the following absolutely convergent, conditionally convergent, convergent?

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+5}$

   (b) $\sum_{n=0}^{\infty} \frac{(-1)^{n-3}\sqrt{n}}{n+4}$

   (c) $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}}$

   (d) $\sum_{n=1}^{\infty} (-1)^n \sin(\pi/n)$

   (e) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

9. **Clicker** The series $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$ converges (why?). What about the following:

$$\sum_{n=0}^{\infty} n\left(\frac{2}{3}\right)^n, \quad \sum_{n=0}^{\infty} n^2\left(\frac{2}{3}\right)^n, \quad \sum_{n=0}^{\infty} n^3\left(\frac{2}{3}\right)^n, \quad \sum_{n=0}^{\infty} n^{10000}\left(\frac{2}{3}\right)^n$$

   (a) None converge

   (b) Some, not all converge

   (c) All converge

10. Let $\sum_{n=0}^{\infty} ar^n$ be a geometric series (so $a_n = ar^n$).

    (a) Compute: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| =$

    (b) Compute: $\lim_{n\to\infty} \sqrt[n]{|a_n|} =$