Math 132 - November 15, 2017

Section 11.4: Comparison Tests

- **Comparison Test For Convergence:** $|a_n| \le |b_n|$, $\sum |b_n|$ converges then $\sum |a_n|$ converges.
- **Comparison Test For Divergence:** $|b_n| \le |a_n|$, $\sum |b_n|$ diverges then $\sum |a_n|$ diverges.
- **Limit Comparison Test 1:** Let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$. If $0 < L < \infty$ then $\sum a_n$ and $\sum b_n$ both converge or both diverge.
- Not in textbook
  Limit Comparison Test 2: Let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$. If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.
- Not in textbook
  Limit Comparison Test 3: Let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$. If $L = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Section 11.5: Alternating Series

Let $S = \sum(-1)^n b_n$ where $b_n > 0$ be an alternating series. S converges if the following are true:

- $b_{n+1} < b_n$ (so the b_n are decreasing)
- $\lim b_n = 0$

And $|R_n| = |S - S_n| \le b_{n+1}$

Section 11.6: Absolute Convergence

- Definition: $\sum a_n$ is **absolutely convergent** if $\sum |a_n|$ converges.
- Definition: $\sum a_n$ is **conditionally convergent** if $\sum a_n$ converges but $\sum |a_n|$ diverges.
- **Theorem:** If $\sum a_n$ is absolutely convergent then $\sum a_n$ is convergent.

1. **Clicker** Fill in the box appropriately (consider n to be a large integer):
   $n^2 + 2n + 1$ ☐ $3n^2$
   (a) $\le$ (b) $\ge$ (c) Neither
2. Converge or Diverge? (use comparison test)
   $\sum_{n=1}^{\infty} \frac{n^2 - 2n - 1}{n^4 + 2n + 1}$ compared to $\sum_{n=1}^{\infty} \frac{1}{n^2}$
3. Converge or Diverge? (use comparison test)
   $\sum_{n=1}^{\infty} \frac{n^2 + 2n + 1}{n^4 - 2n - 1}$ compared to $\sum_{n=1}^{\infty} \frac{1}{n^2}$
4. **Clicker** Let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.
   If $L = 1$, which of the following do you expect:
   (a) $a_n < b_n$ (b) $a_n = b_n$ (c) $a_n > b_n$ (d) None of these
5. **Clicker** Let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.
   If $L = 0$, which of the following do you expect:
   (a) $a_n < b_n$ (b) $a_n = b_n$ (c) $a_n > b_n$ (d) None of these
6. **Clicker** Let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.
   If $L = \infty$, which of the following do you expect:
   (a) $a_n < b_n$ (b) $a_n = b_n$ (c) $a_n > b_n$ (d) None of these
7. Converge or Diverge? (do a limit comparison test)
   $\sum_{n=1}^{\infty} \frac{n^2 - 2n - 1}{n^4 + 2n + 1}$ compared to $\sum_{n=1}^{\infty} \frac{1}{n^2}$
8. Converge or Diverge? (do a limit comparison test)
   $\sum_{n=1}^{\infty} \frac{n^2 + 2n + 1}{n^4 + 2n - 1}$ compared to $\sum_{n=1}^{\infty} \frac{1}{n^2}$

9. Determine convergence of the following. Use a comparison or limit comparison (try to use both!).

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n-1}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$

10. **Alternating Series:** Does the series converge or diverge:
    $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \cdots$

11. **Clicker** Converge or Diverge? $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

    If it converges, find the sum accurate to 0.01.

    (a) Converge (b) Diverge

12. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$.

    (a) $S_{15} = 0.8245417573\ldots$.
    How close does the alternating series remainder theorem guarantee S_{15} to be from S?

    (b) If you want the partial sum to be within 0.0001 of the sum, how many terms do you need to take?

13. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$.

    (a) $S_{15} = 0.901676118\ldots$.
    How close does the alternating series remainder theorem guarantee S_{15} to be from S?

    (b) If you want the partial sum to be within 0.001 of the sum, how many terms do you need to take?

14. **Clicker** Does the series converge absolutely: $\displaystyle\sum_{i=1}^{\infty} \frac{(-1)^{n+1}}{n^2+2n+1}$

    (a) Yes (b) No (c) It converges conditionally

15. Are the following absolutely convergent? Are they conditionally convergent?

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+5}$

    (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-3}\sqrt{n}}{n+4}$

    (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}}$

    (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin(\pi/n)$

    (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$