Math 132 - November 13, 2017 - Exam 3 review
Solutions

Topics for Exam 3		
7.3 Trig Sub	7.4: Partial Fracs	7.5: Integration
7.8: Improper	8.1: Length	8.2: Area
11.1: Sequences	11.2: Geometric, Telescoping	11.3: Integral Test

1. $\int_3^\infty \frac{1}{(x-1)^3}\,dx = 1/8$
2. $\int_1^\infty \frac{1}{x}\,dx =$ Diverges
3. $\int_1^\infty \frac{1}{\sqrt{x}}\,dx =$ Diverges
4. $\int_1^\infty \frac{1}{x^{3/2}}\,dx = 2$
5. $\int_1^\infty \frac{1}{x^{1/4}}\,dx =$ divergent
6. $\int_0^5 \frac{1}{(x-2)}\,dx =$ Diverges
7. $\int_1^2 \frac{3x+2}{x^2+x}\,dx = \ln 6$
8. $\int_1^2 \frac{3x^2+3x+1}{x(x+1)^2}\,dx = \ln(9/2) - 1/6$
9. $\int_0^{1/\sqrt{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx = \frac{\pi-2}{8}$
10. $\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx = \ln(1+\sqrt{2})$
11. $\int_0^1 \frac{4x^2+2x+4}{(x^2+1)^2}\,dx = \pi + 1/2$
12. Find $\int_7^8 \frac{1}{(8-x)^{1/3}}\,dx = \frac{3}{2}$
13. $\int_1^\infty \frac{x^2}{(2+x^3)^2}\,dx = \frac{1}{9}$
14. $\int_2^3 \frac{3x+3}{(x+2)(x-1)}\,dx = \ln 5$
15. $\int_0^5 \frac{x^2}{x^2+25}\,dx = 5 - 5\pi/4$
16. $\int_1^2 \frac{2x^2-3x+2}{x(x^2+1)}\,dx = A\arctan(2) + B\ln 2 + C\pi$. Find A, B, C
Solution: $A = -3$, $B = 2$, $C = 3/4$
17. Find the form of partial fractions for $\frac{x+1}{x^4+x^2}$.
Solution: $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$
18. Find the length of $y = \frac{4}{3}x + \frac{5}{3}$ over $-1 \le x \le 2$.
Solution: 5
19. A line segment with end points $(2, 3)$ and $(6, 0)$ is rotated about the y-axis. What is the surface area?
Solution: 40π
20. The graph of $y = 3\sqrt{x}$ for $4 \le x \le 10$ is rotated about the x-axis. What is the surface area?
Solution: 109π
21. Find partial fractions decomposition for $\frac{5x^2-5x-6}{x^3-2x^2-3x} = \frac{2}{x} + \frac{1}{x+1} + \frac{2}{x-3}$
22. $\int \frac{2x^3+3x^2+3x+1}{x^3+x}\,dx = 2x + \ln|x^3+x| + \arctan x + C$
23. $\int_2^4 \frac{dt}{t\sqrt{t^2-4}}\,dt = \pi/6$
24. Find length of graph $y = (1/3)(x^2+2)^{3/2}$ over $0 \le x \le 3$.
Solution: 12

25. Find surface area of $y = x + 1$ rotated about x axis over the interval $0 \le x \le 1$.
Solution: $3\sqrt{2}\pi$

26. Find the integral that represents surface area of revolution of $y = e^x$ over $0 \le x \le 1$, rotated about the x axis.
Solution: $2\pi \int_0^1 e^x \sqrt{1 + e^{2x}}\, dx = 2\pi \int_1^e \sqrt{1 + u^2}\, du$

27. Find the limit

(a) $\lim_{n\to\infty} \frac{4n-3}{3n-4} = 4/3$
(b) $\lim_{n\to\infty} \left(\frac{n+1}{2n}\right)\left(1 - \frac{1}{n}\right) = \frac{1}{2}$
(c) $\lim_{n\to\infty} \frac{1}{(1/10)^n} =$ Diverges
(d) $\lim_{n\to\infty} (\cos(1/n) + n\sin(1/n)) = 2$
(e) $\lim_{n\to\infty} n^2 e^{-5n} = 0$
(f) $\lim_{n\to\infty} \left(1 + \frac{4}{n}\right)^n = e^4$

28. Find the sum, if possible

(a) $\frac{4}{9} + \frac{4}{27} + \frac{4}{81} + \frac{4}{243} + \cdots = 2/3$
(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{n+1} - \frac{3}{n+2}\right) = 3/2$
(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^{n-1}} = 16$
(d) $3 + \frac{6}{5} + \frac{12}{25} + \frac{24}{125} + \frac{48}{625} + \cdots = 5$
(e) $\displaystyle\sum_{n=1}^{\infty} \frac{8}{3^n} = 4$
(f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2^n} - \frac{1}{4^{n-1}}\right) = -1/3$
(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n} = \frac{1}{4}$
(h) $\displaystyle\sum_{n=2}^{\infty} 3\left(\frac{-1}{2}\right)^n = 1/2$
(i) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{(n+1)^2} - \frac{1}{n^2}\right) = -1$
(j) $\displaystyle\sum_{n=1}^{\infty} (\cos(1/n) - \cos(1/(n+1))) = -1 + \cos 1$
(k) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n+2} - \frac{1}{n+7}\right) = 153/140$

29. Converge or diverge?

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ **Solution:** Converge
(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ **Solution:** Diverge
(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$ **Solution:** Diverge
(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{n^{3/2}}$ **Solution:** Diverge
(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$ **Solution:** Converge
(f) $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{15}$ **Solution:** Diverges
(g) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{n^3 - 1}$ **Solution:** Diverges

30. If $a_n = n^2 e^{-5n}$, find $\lim_{n\to\infty} a_n$.
Solution: 0

31. Find $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2^n} - \frac{1}{4^{n-1}}\right)$
Solution: $-1/3$

32. Find $\displaystyle\int \frac{x^3 - 3x^2 - 9}{x^3 - 3x^2}\, dx$
Solution: $\ln|x| - \ln|x - 3| + x - 3x^{-1} + C$

33. Find $\displaystyle\int_0^5 \frac{x^2}{\sqrt{25 - x^2}}\, dx$
Solution: $25\pi/4$

34. Find $\displaystyle\int_{-2}^{1} \frac{1}{x^2+4x+13}\,dx$

Solution: $\pi/12$

35. Find $\displaystyle\lim_{n\to\infty} \frac{n^4}{n!}$

Solution: 0

36. Converge or diverge: $\displaystyle\int_0^{\infty} \frac{1}{x^2+\sqrt{x}}\,dx$

Solution: Converges. You have to split it into two integrals.

$\displaystyle\int_1^{\infty} \frac{1}{x^2+\sqrt{x}}\,dx \le \int_1^{\infty} \frac{1}{x^2}\,dx$

$\displaystyle\int_0^{1} \frac{1}{x^2+\sqrt{x}}\,dx \le \int_0^{1} \frac{1}{\sqrt{x}}\,dx$

These converge, so therefore the sum converges.