Math 132 - November 1, 2017

4.4 L'Hopital's Rule

- $\left(\frac{0}{0}\right)$: If $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = 0$ then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$
- $\left(\frac{\infty}{\infty}\right)$: If $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$ then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$

List of Indeterminate Forms:
$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty$, $\infty-\infty$, 0^0, 1^∞, ∞^0.
List of Indeterminate Forms handled by L'Hopital: $\frac{0}{0}$, $\frac{\infty}{\infty}$

11.1 Sequences

Terms

- Increasing sequence
- Decreasing Sequence
- Monotonic Sequence
- Bounded Above Sequence
- Bounded Below Sequence
- Bounded Sequence

Limit Properties

- $\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$
- $\lim_{n\to\infty} c\cdot a_n = c\cdot \lim_{n\to\infty} a_n$
- $\lim_{n\to\infty} a_n\cdot b_n = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$
- $\lim_{n\to\infty} c = c$
- $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$
- $\lim_{n\to\infty} a_n^p = \left(\lim_{n\to\infty} a_n\right)^p$
- If $\lim a_n = \lim c_n = L$ and $a_n < b_n < c_n$ then $\lim b_n = L$
- If $f(n) = a_n$ and $\lim_{x\to\infty} f(x) = L$ then $\lim a_n = L$

Warm-up Problems

1. **Clicker** Which limits below are indeterminant forms? Compute the limits using L'Hopital's Rule.
   (a) $\lim_{x\to 4} \frac{\sin x}{x}$
   (b) $\lim_{x\to 0} \frac{\sin x}{x}$
   (c) $\lim_{x\to 0} \frac{\cos x}{x}$
   (d) $\lim_{x\to\infty} xe^{-x}$
2. Set up arc length integral (compute if you have time!) for $y = \ln\cos(x)$ on $[0, \pi/4]$.
3. Set up surface area integral (compute if you have time!).
   Rotate the arc $y = x^2$ between $x = 1$ to $x = 2$ about the line $x = -4$.

Class Problems

4. Use L'Hopital's Rule, if necessary. Make sure you identify the form of the limit!
   (a) $\lim_{x\to 0} \frac{e^x - 1}{2x}$
   (b) $\lim_{x\to\infty} \frac{e^x - 1}{2x}$
   (c) $\lim_{x\to 0} \frac{e^x}{2x}$
   (d) $\lim_{x\to\infty} \frac{e^x}{2x}$

5. **Clicker** Find the next term in the sequence

$$1, 2, 3, \ldots$$

(a) 0 (b) 1 (c) 4 (d) 42

6. **Clicker** Find the next term in the sequence

$$1, 1, 2, 3, 5, 8, \ldots$$

(a) 9 (b) 10 (c) 11 (d) 12 (e) 13

7. **Clicker** Find the next term in the sequence

$$157.41, \quad 163.05, \quad 166.72, \quad 168.97, \quad \ldots$$

(a) 151.12 (b) 163.32 (c) 171.21 (d) 175.21 (e) 181.98

8. For each of the sequences, write the first few terms of the sequence and then find $\lim_{n\to\infty} a_n$

(a) $a_n = 2 + (0.1)^n$
(b) $a_n = \frac{n+(-1)^n}{n}$
(c) $a_n = \frac{1-2n}{1+2n}$
(d) $a_n = \sin n$
(e) $a_n = \sin(1/n)$
(f) $a_n = \cos(1/n)$

9. Trickier sequences. Write the first few terms of the sequence and then find $\lim_{n\to\infty} a_n$

(a) $a_n = \frac{\ln n}{n}$
(b) $a_n = \sqrt[n]{n}$
(c) Let $x > 0$ be fixed. Let $a_n = x^{1/n}$.
(d) Let x be fixed and let $a_n = x^n$.
(e) Let x be fixed and let $a_n = \left(1 + \frac{x}{n}\right)^n$.
(f) Let x be fixed and let $a_n = \frac{x^n}{n!}$.
(g) $a_n = \frac{n!}{n^n}$