Math 132 - October 20, 2017

7.4 Partial Fractions

Partial Fraction Steps:

1. Long Division (if deg top > deg bottom)
2. Factor the denominator (linear and quadratic irreducibles)
3. Determine the "form" (linear, quadratic, repeated)
4. Solve for constants
5. Integrate term by term

Warm-up Problems

1. **Clicker** Solve the partial fraction decomposition

$$\frac{x^2+x+2}{(x-2)(x-1)(x+1)} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{x+1}$$

(a) $C = 1$ (b) $C = \frac{1}{3}$ (c) $C = -2$ (d) $C = \frac{8}{3}$

2. Integrate $\displaystyle\int \frac{x^5 - 5x^3 + x^2 + 5x + 2}{x^3 - 2x^2 - x + 2}\, dx$

Note: You have to do every step in this problem, so probably not reasonable to complete this quickly. But it would probably be worth your time to work on this problem after class.

$$\begin{aligned}\int \frac{x^5 - 5x^3 + x^2 + 5x + 2}{x^3 - 2x^2 - x + 2}\, dx &= \int \left(x^2 + 2x + \frac{x^2+x+2}{(x-1)(x-2)(x+1)} \right) dx \\ &= \int \left(x^2 + 2x + \frac{1}{3(x+1)} - \frac{2}{x-1} + \frac{8}{3(x-2)} \right) dx \\ &= \frac{1}{3}x^3 + x^2 + \frac{1}{3}\ln|x+1| - 2\ln|x-1| + \frac{8}{3}\ln|x-2| + C\end{aligned}$$

Class Problems

Note: Problems 3 through 8 are linked (corresponding parts correspond).

3. Long division practice:

(a) $\dfrac{6x^4 + 3x^3 - 172x^2 + 102x - 329}{x^2 + x - 30}$

(b) $\dfrac{3x^5 - 2x^4 + 13x^3 - 8x^2 + 5x + 1}{x^3 + 4x}$

(c) $\dfrac{x^8 + 4x^6 - x^5 - 4x^3 + x + 1}{x^5 + 4x^3}$

(d) $\dfrac{2x^5 - 4x^3 + 3x^2 + 3x + 1}{x^4 - 2x + 1}$

(e) $\dfrac{2x^6 - 2x^5 + 20x^4 - 20x^3 + 21x^2 - 17x + 1}{x^6 - x^5 + 10x^4 - 10x^3 + 9x^2 - 9x}$

4. Factoring Practice:

(a) $x^2 + x - 30$

(b) $x^3 + 4x$

(c) $x^5 + 4x^3$

(d) $x^4 - 2x^2 + 1$

(e) $x^6 - x^5 + 10x^4 - 10x^3 + 9x^2 - 9x$

5. Writing down partial fraction form

   (a) $\dfrac{x+1}{(x-5)(x+6)}$

   (b) $\dfrac{x+1}{x(x^2+4)}$

   (c) $\dfrac{x+1}{x^3(x^2+4)}$

   (d) $\dfrac{3x^2+x+1}{(x-1)^2(x+1)^2}$

   (e) $\dfrac{3x^2+x+1}{x(x-1)(x^2+9)(x^2+1)}$

6. Find the system of equations to solve.

   (a) $\frac{x+1}{(x-5)(x+6)} = \frac{A}{x-5} + \frac{B}{x+6}$

   (b) $\frac{x+1}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

   (c) $\frac{x+1}{x^3(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+4}$

   (d) $\frac{3x^2+x+1}{(x-1)^2(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2}$

   (e) $\frac{3x^2+x+1}{x(x-1)(x^2+9)(x^2+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{x^2+9}$

7. Solve the system of equations

   (a) $6A - 5B = 1$, $A + B = 1$

   (b) $A + B = 0$, $C = 1$, $4A = 1$

   (c) $A + D = 0$, $B + E = 0$, $4A + C = 0$, $4B = 1$, $4C = 1$

   (d) $A + C = 0$, $A + B - C + D = 3$, $-A + 2B - C - 2D = 1$, $-A + B + C + D = 1$

   (e) $A + B + C + E = 0$, $-A - C + D - E + F = 0$, $10A + 10B + 9C - D + E - F = 0$, $-10A - 9C + 9D - E + F = 3$, $9A + 9B - 9D - F = 1$, $-9A = 1$

8. Integrate

   (a) $\int \left(6x^2 - 3x + 11 + \frac{6}{11(x-5)} + \frac{5}{11(x+6)}\right)\, dx$

   (b) $\int \left(3x^2 - 2x + 1 + \frac{1}{4x} - \frac{x}{4(x^2+4)} + \frac{1}{x^2+4}\right)\, dx$

   (c) $\int \left(x^3 - 1 - \frac{1}{16x} + \frac{1}{4x^2} + \frac{1}{4x^3} + \frac{x}{16(x^2+4)} - \frac{1}{4(x^2+4)}\right)\, dx$

   (d) $\int \left(2x + \frac{1}{2(x-1)} + \frac{5}{4(x-1)^2} - \frac{1}{2(x+1)} + \frac{3}{4(x+1)^2}\right)\, dx$

   (e) $\int \left(2 - \frac{1}{9x} + \frac{1}{4(x-1)} - \frac{3x}{16(x^2+1)} + \frac{1}{16(x^2+1)} + \frac{7x}{144(x^2+9)} - \frac{5}{16(x^2+9)}\right)\, dx$

9. Determine the methods to integrate the following (simplify integrand, if necessary).

   (a) $\displaystyle\int e^{x+e^x}\, dx$

   (b) $\displaystyle\int t \sec t \tan t\, dt$

   (c) $\displaystyle\int \frac{2x-3}{x^3+3x}\, dx$

   (d) $\displaystyle\int \frac{\ln(\tan x)}{\sin x \cos x}\, dx$

   (e) $\displaystyle\int \frac{1+\tan x}{\sec x - 1}\, dx$

   (f) $\displaystyle\int \sqrt{3-2x-x^2}\, dx$

   (g) $\displaystyle\int \frac{x^2}{x^2+1}\, dx$

   (h) $\displaystyle\int \frac{\cos(\frac{1}{x})}{x^3}\, dx$

   (i) $\displaystyle\int \frac{\sin^3 x}{\cos x}\, dx$

   (j) $\displaystyle\int \frac{\tan^{-1}(x)}{x^2}\, dx$