Math 132 - October 18, 2017

7.4 Partial Fractions

Partial Fraction Steps:

1. Long Division (if deg top > deg bottom)
2. Factor the denominator
   (linear and quadratic irreducibles)
3. Determine the "form" (linear, quadratic, repeated)
4. Solve for constants
5. Integrate term by term

Warm-up Problems

1. **Clicker** Solve the system of equations

$$\begin{aligned} 2A + 2B + C &= 13 \\ 3A - B + 2C &= -6 \\ -2A &= 1 \end{aligned}$$

   $B =$??

   (a) $-1/2$
   (b) $13/2$
   (c) 1

2. Find the general integrals below. (These are the types of integrals that should appear when partial fractions is conducted correctly.)

   (a) $\displaystyle\int \frac{1}{ax+b}\,dx$
   (b) $\displaystyle\int \frac{1}{(ax+b)^2}\,dx$
   (c) $\displaystyle\int \frac{x}{x^2+a^2}\,dx$
   (d) $\displaystyle\int \frac{x}{(x^2+a^2)^2}\,dx$
   (e) $\displaystyle\int \frac{1}{x^2+a^2}\,dx =$
   (f) $\displaystyle\int \frac{1}{(x^2+a^2)^2}\,dx$

Class Problems

3. **Clicker** Determine the correct partial fraction form. How many constants are needed? $\int \frac{1}{(x-1)(x-2)}\,dx$

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

4. **Clicker** Determine the correct partial fraction form. How many constants are needed? $\int \frac{1}{(x-1)(x-2)^2}\,dx$

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

5. **Clicker** Determine the correct partial fraction form. How many constants are needed? $\int \frac{1}{(x-1)(x^2+4)}\,dx$

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

6. **Clicker** Determine the correct partial fraction form. How many constants are needed? $\int \frac{2}{(x-1)(x^2+4)^2}\,dx$

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

7. Determine the correct form for the following

   (a) $\int \frac{x+3}{x(3x+2)}\,dx$

   (b) $\int \frac{x+3}{x(3x+2)^3}\,dx$

   (c) $\int \frac{x+3}{(3x+2)^3}\,dx$

   (d) $\int \frac{3}{(3x+2)^3}\,dx$

   (e) $\int \frac{x^2+x+1}{x(x^2+9)}\,dx$

   (f) $\int \frac{x^2+x+1}{x(x^2+9)(x^2+4)}\,dx$

8. For the previous problems, solve for the constants and compute the integrals.