Math 132 - October 2, 2017
Solutions

Sections Covered: 7.1

Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

Warm-up Problems

1. **Clicker** The integral below can be done with integration by parts:

$$\int (\ln x)^2\,dx$$

What is a good choice for u?

(a) 1
(b) $\ln x$
(c) $(\ln x)^2$ **Correct**
(d) dx
(e) It can't be done by integration by parts.

Solution: If $u = (\ln x)^2$ then $du = 2(\ln x)\cdot\frac{1}{x}$. $dv = dx$ and $v = x$ and the integral becomes

$$\begin{aligned}\int (\ln x)^2\,dx &= x(\ln x)^2 - \int 2\ln x\cdot\frac{1}{x}\cdot x\,dx\\ &= x(\ln x)^2 - \int 2\ln x\,dx\end{aligned}$$

Integration by parts is necessary a second time. You should get:
$x(\ln x)^2 - 2x\ln x + 2x + C$

2. In Problem 1, what is v?

Solution: x.

Class Problems

3. $\int e^x\cos x\,dx$

Solution: Trickier, use integration by parts twice and notice the pattern.
$\frac{1}{2}e^x(\cos x + \sin x) + C$

The trick for these is the following:

$$\begin{aligned}I =& \int e^x\cos x\,dx \qquad (u = e^x, du = \cos x\,dx)\\ =& e^x\sin x - \int e^x\sin x\,dx \qquad (u = e^x, dv = \sin x\,dx)\\ =& e^x\sin x + e^x\cos x - \int e^x\cos x\,dx\\ I =& e^x\sin x + e^x\cos x - I\\ 2I =& e^x\sin x + e^x\cos x\\ I =& \frac{1}{2}(e^x\sin x + e^x\cos x)\end{aligned}$$

Be sure to try this again but with $u = \cos x$ and $dv = e^x\,dx$. It should work just the same.

4. $\int e^x \sin x \, dx$

   Solution: Integrate by parts twice:
   $I = -e^x \cos x + \int e^x \cos x \, dx = -e^x \cos x + \left(e^x \sin x - \int e^x \sin x \, dx\right)$
   $I = \frac{1}{2}(-e^x \cos x + e^x \sin x)$

5. $\int e^{3x} \cos 4x \, dx$

   Solution: $\frac{1}{25}e^{3x}(4\sin 4x + 3\cos 4x) + C$

6. $\int x^2 e^{3x} \, dx$

   Solution: $\frac{1}{27}e^{3x}(9x^2 - 6x + 2) + C$

7. $\int x^3 e^{3x} \, dx$

   Solution: $\frac{1}{27}e^{3x}(9x^3 - 9x^2 + 6x - 2) + C$

8. $\int x \sin x \cos x \, dx$

   Solution: One option is to use $u = x$ and $dv = \sin x \cos x \, dx$. Another method is to use a trig identity and transform $\sin x \cos x = \frac{1}{2}\sin 2x$ and then proceed as normal. In any case your solution might look like one of the following:
   $\frac{1}{4}x\sin^2 x - \frac{1}{4}x\cos^2 x + \frac{1}{8}\sin 2x + C$ OR
   $\frac{1}{8}\sin 2x - \frac{1}{4}x\cos 2x + C$

9. $\int x \ln x \, dx$

   Solution: Use $u = \ln x$ and $dv = x \, dx$.
   $\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

10. $\int \frac{\ln x}{x^5} \, dx$.

    Solution: You can do a substitution of $w = \ln x$ and transform into a w integral (TRY IT!). Or you can use $u = \ln x$ and $dv = \frac{1}{x^5} \, dx$.
    $\int \frac{\ln x}{x^5} \, dx = -\frac{\ln x}{4x^4} - \frac{1}{16x^4} + C$

11. $\int \arctan x \, dx$

    Solution: use parts with $u = \arctan x$ and $dv = dx$.

$$\begin{aligned}\int \arctan x \, dx &= x \arctan x - \int \frac{x}{x^2+1} \, dx \\ &= x \arctan x - \frac{1}{2}\ln(x^2+1) + C\end{aligned}$$

12. $\int 2x \arctan x \, dx$

    Solution: There is an algebraic trick at the end (a method which we will formalize later as **Partial Fractions**.
    Use parts with $u = \arctan x$ and $dv = 2x \, dx$.

$$\begin{aligned}\int 2x \arctan x \, dx =& x^2 \arctan x - \int \frac{x^2}{x^2+1} \, dx \\ =& x^2 \arctan x - \int \frac{x^2+1-1}{x^2+1} \, dx \\ =& x^2 \arctan x - \int \frac{x^2+1}{x^2+1} \, dx + \int \frac{1}{x^2+1} \, dx \\ =& x^2 \arctan x - x + \arctan x + C\end{aligned}$$

13. **Clicker** Choose the correct u if using integration by parts: $\int (\text{Polynomial}(x))e^x \, dx$

    (a) $u = 1$ (b) $u = \text{Polynomial}(x)$ **Correct** (c) $u = e^x$ (d) $u = \sin x$ (e) $u = \cos x$

14. **Clicker** Choose the correct u if using integration by parts: $\int (\text{Polynomial}(x)) \sin x \, dx$

    (a) $u = 1$ (b) $u = \text{Polynomial}(x)$ **Correct** (c) $u = e^x$ (d) $u = \sin x$ (e) $u = \cos x$

15. **Clicker** Choose the correct u if using integration by parts: $\int (\text{Polynomial}(x)) \cos x \, dx$

(a) $u = 1$ (b) $u = \text{Polynomial}(x)$ **Correct** (c) $u = e^x$ (d) $u = \sin x$ (e) $u = \cos x$

16. **Clicker** Choose the correct u if using integration by parts: $\int e^x \sin x \, dx$

(a) $u = 1$ (b) $u = e^x$ **Correct** (c) $u = \sin x$ **Correct** (d) $u = \cos x$ (e) $u = \ln x$

17. **Clicker** Choose the correct u if using integration by parts: $\int e^x \cos x \, dx$

(a) $u = 1$ (b) $u = e^x$ **Correct** (c) $u = \sin x$ (d) $u = \cos x$ **Correct** (e) $u = \ln x$

18. **Clicker** Choose the correct u if using integration by parts: $\int \ln x \, dx$ (a) $u = 1$ (b) $u = e^x$ (c) $u = \sin x$ (d) $u = \cos x$ (e) $u = \ln x$ **Correct**

Lecture Notes: Integration by parts, the "normal" types you will see and you have to have "down cold":

- $\int (\text{Polynomial}(x)) e^x \, dx$
- $\int (\text{Polynomial}(x)) \sin x \, dx$
- $\int (\text{Polynomial}(x)) \cos x \, dx$
- $\int e^x \sin x \, dx$
- $\int e^x \cos x \, dx$
- $\int \ln x \, dx$

19. $\int \sin x \cos x \, dx$

Solution: This is a simple substitution. You can use either $u = \sin x$ or $u = \cos x$. In either case you get

$$\begin{aligned}
\int \sin x \cos x \, dx &= \int u \, du \qquad (u = \sin x) \\
&= \frac{u^2}{2} + C = \frac{1}{2} \sin^2 x + C \\
\int \sin x \cos x \, dx &= -\int u \, du \qquad (u = \cos x) \\
&= -\frac{u^2}{2} + C = -\frac{1}{2} \cos^2 x + C
\end{aligned}$$

How can this be????