Math 132 - September 8, 2017
Solutions

5.3, 5.4: FTC

I. $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = f(x)$

II. $\displaystyle\int_a^b F'(x)\,dx = F(b) - F(a)$

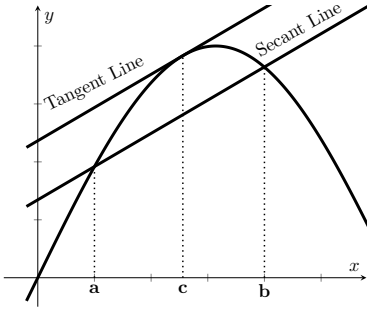
Warm-up Problems

1. **Clicker** Find the derivative

$$\frac{d}{dx}\int_{-x^2}^{e^x} \sin(t^2)\,dt$$

   (a) $e^x \sin(e^{2x}) - 2x\sin(x^4)$
   (b) $e^x \sin(e^{2x}) + 2x\sin(x^4)$ **Correct**
   (c) $\sin(e^{2x}) - \sin(x^4)$
   (d) $e^x \sin(e^{2x})$
   (e) I have no idea

2. Explain the MVT:



Class Problems

Lecture Notes: The FTC part 1 essentially says that the derivative of an integral is equal to the function. What about the integral of a derivative? This turns out to be very useful: FTC, part 2.
Here is the set up. We have a function $F(x)$ and its derivative $F'(x)$. We want to compute

$$\int_a^b F'(x)\,dx$$

We do this by doing a Riemann Sum with sample points given by the MVT.
Back to the FTC part 2. We have a function $F(x)$ and its derivative $F'(x)$. We want to compute

$$\int_a^b F'(x)\,dx$$

We do this by doing a Riemann Sum with sample points given by the MVT.
What this means is the following. We compute a Riemann Sum and take the limit:

$$R_n = \sum_{i=1}^{n} F'(x_i^*)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and x_i^* is chosen so that

$$F'(x_i^*) = \frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}} = \frac{F(x_i) - F(x_{i-1})}{\Delta x}$$

Plugging this in:

$$R_n = \sum_{i=1}^{n} F'(x_i^*)\Delta x = \sum_{i=1}^{n} \left(\frac{F(x_i) - F(x_{i-1})}{\Delta x} \right) \Delta x$$
$$= \sum_{i=1}^{n} (F(x_i) - F(x_{i-1})) = F(b) - F(a)$$

Now take the limit as $n \to \infty$ and we get the result. (Notice that our cleverness makes it so we didn't have to take the limit.)
Note on the proof above: You are not required or expected to recreate this, but it is a nice proof of the FTC part 2. You are expected to be able to use the FTC part 2 and I believe that seeing why it is true is helpful.
The point of FTC part 2 is that to compute definite integrals, you just compute antiderivatives (indefinite integrals).

3. Find the following indefinite integrals using FTC part 2.

   (a) $\int_{-4}^{12} dx = 16$

   (b) $\int_{1}^{9} -2\, dx = -16$

   (c) $\int_{1}^{4} x\, dx = 15/2$

   (d) $\int_{-1}^{1} x^2\, dx = 2/3$

   (e) $\int_{0}^{\pi} \sin x\, dx = 2$

   (f) $\int_{1}^{10} \frac{1}{x}\, dx = \ln 10$

   (g) $\int_{0}^{\pi/4} \sec^2 x\, dx = 1$

   (h) $\int_{1}^{4} x\, dx = 15/2$

Lecture Notes: If we accept that finding areas is useful and something we want to do then that means we will want to use the FTC (part 2). Thus, we need to be able to compute anti-derivatives. We have rules for computing derivatives (and everyone should be able to take the derivative of just about anything).

Goal: be able to find an antiderivative of just about anything.

4. **Clicker**
   (True/False) Any derivative formula gives a corresponding indefinite integral formula?
   (a) True **Correct** (b) False (c) Neither True or False

Review Problems

5. Suppose $\int_1^3 f(x)\, dx = 6$ and $\int_1^3 g(x)\, dx = 2$. What is $\int_1^3 (2f(x) - 3g(x))\, dx$?
   Solution: 6

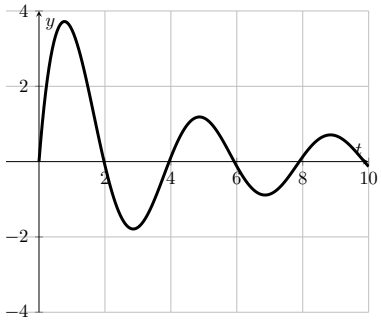
6. Suppose $\int_1^4 f(x) = 8$, $\int_1^2 f(x) = 4$, $\int_3^4 f(x) = 2$, what is $\int_2^3 f(x) = 8$?
   Solution: 2

7. Find $\int_0^1 x\,(\sqrt{x} + \sqrt[3]{x})\, dx$
   Solution: 29/35

8. If $g(x) = \int_1^{\sqrt{x}} \sin(t^2)\, dt$, find $g'(x)$.

Solution: $\dfrac{\sin x}{2\sqrt{x}}$

9. **Clicker** Let $g(x) = \int_0^x f(t)\, dt$ where $f(t)$ is the graph below. Determine which of the statements are true:



(a) g attains an absolute maximum at $x = 2$ **Correct**

(b) g has a local maximum at $x = 5$

(c) g has a local minimum at $x = 4$ **Correct**

(d) g is concave down on $[0, 2]$