Math 131 - April 22, 2016
Solutions

Warm-up Problems

The *Definition* of the definite integral is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

This represents the "signed area" under the curve.

1. Suppose $\int_1^5 f(x)\,dx = 13$. Find the following, justify your answers.

   (a) $\int_1^5 2\,f(x)\,dx = 26$

   (b) $\int_5^1 f(x)\,dx = -13$

   (c) $\int_1^4 f(x)\,dx + \int_4^5 f(x)\,dx = 13$

Lecture Problems

2. Compute exactly, using the FTC.

   (a) $\int_1^3 x^2 - 2x\,dx = 2/3$

   (b) $\int_0^{\pi/2} \sin x + 4\,dx = 2\pi + 1$

   (c) $\int_1^2 \frac{1}{x}\,dx = \ln 2$

   (d) $\int_2^{10} \frac{1}{x}\,dx = \ln 10 - \ln 2$

3. Find an explicit formula (without an integral) for the following integral functions

   (a) $g(x) = \int_1^x \sin t\,dt = -\cos x + \cos 1$

   (b) $g(x) = \int_1^x \cos t\,dt = \sin x - \sin 1$

   (c) $g(x) = \int_1^{x^2} \cos t\,dt = \sin x^2 - \sin 1$

   (d) $g(x) = \int_1^x \frac{1}{t}\,dt = \ln x$

   (e) $g(x) = \int_3^x \frac{1}{t}\,dt = \ln x - \ln 3$

   (f) $g(x) = \int_3^{x^2+4} \frac{1}{t}\,dt = \ln(x^2+4) - \ln 3$

   (g) $g(x) = \int_x^{x^2+4} \frac{1}{t}\,dt = \ln(x^2+4) - \ln x$

4. For each of the integral functions in Problem 3, compute $g'(x)$.