Math 131 - March 25, 2016
Solutions

Warm-up Problems

1. What is an increasing function?
   (If you don't know, just make something up that makes sense!)

   Solution: A function f is increasing if, wheneven $x_1 < x_2$ we also have $f(x_1) < f(x_2)$.

2. What is a decreasing function?
   (If you don't know, just make something up that makes sense!)

   Solution: A function f is decreasing if, wheneven $x_1 < x_2$ we also have $f(x_1) > f(x_2)$.

Lecture Problems

3. For the given functions, find the regions where the function is increasing and decreasing. Identify local max and mins.

   (a) $f(x) = x^3 - 9x^2 + 15x + 7$ **Solution:** $f'(x) = 3(x-1)(x-5)$

   (b) $f(x) = x^4 - 8x^3 + 10x^2 + 1$. **Solution:** $f'(x) = 4x(x-5)(x-1)$

   (c) $f(x) = 6x^5 - 45x^4 + 50x^3 + 7$ **Solution:** $f'(x) = 30x^2(x-5)(x-1)$

4. Draw a graph that matches the given data

$f'(x) > 0$
$f''(x) < 0$

5. Draw a graph that matches the given data

$f'(x) > 0$ when $x < 3$ $f'(x) < 0$ when $x > 3$
$f''(x) < 0$

6. Draw a graph that matches the given data

$f'(x) > 0$ when $x < 3$ $f'(x) < 0$ when $x > 5$ $f'(x) > 0$ when $x < 7$
$f''(x) < 0$ when $x < 4$ $f''(x) > 0$ when $x > 4$

7. Draw a graph that matches the given data

$f'(x) < 0$ when $x < 3$ $f'(x) > 0$ when $x > 5$ $f'(x) < 0$ when $x < 7$
$f''(x) > 0$ when $x < 4$ $f''(x) < 0$ when $x > 4$

8. Draw a graph that matches the given data

$f'(x) > 0$ when $x < 2$
$f'(x) > 0$ when $2 < x < 4$
$f'(x) < 0$ when $4 < x < 6$
$f'(x) < 0$ when $x > 6$
$f''(x) < 0$ when $x < 2$
$f''(x) > 0$ when $2 < x < 3$
$f''(x) < 0$ when $3 < x < 5$
$f''(x) > 0$ when $5 < x < 6$
$f''(x) < 0$ when $x > 6$

9. Draw a graph that matches the given data

$f'(x) > 0$ when $x < 2$
$f'(x) > 0$ when $2 < x < 4$
$f'(x) < 0$ when $4 < x < 6$
$f'(x) < 0$ when $x > 6$
$f''(x) > 0$ when $x < 2$
$f''(x) > 0$ when $2 < x < 3$
$f''(x) > 0$ when $3 < x < 5$
$f''(x) < 0$ when $5 < x < 6$
$f''(x) > 0$ when $x > 6$

10. Let $f(x) = x^3 - 3x^2 - 9x + 3$

    (a) Find all critical points of f. **Solution:** $f'(x) = 3(x-3)(x+1)$

    (b) Find all possible inflection points of f. **Solution:** $f''(x) = 6x - 6$

    (c) Find all intervals where f is increasing and decreasing.

    (d) Find all intervals where f is concave up and down.

    (e) Find all local max and mins of f.

    (f) Find all inflection points of f.

    (g) Draw a nice graph of f.