

Student Involvement: For each step of each problem:

- Write down ways you can involve the students (questions you can ask).
- Write down which parts of the problem will be difficult for the students and how to handle these.
- Write down variations that the students will suggest (and how you can handle these suggestions). (What is the “best” way to solve the problem?)
- Think about what to do if the students are just plain stuck or don’t want to be involved.
- Think about what mathematics the students will need help with and how to present this.
- Think about the “cookbook” solution to these problems and how to deal with this.

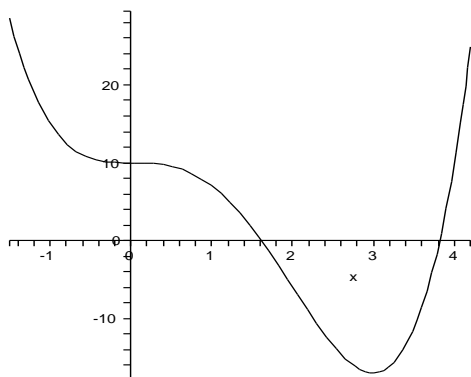
1. Find the derivative of $y = \sin^3(x^3 + \ln x)$.

Solution:

$$\begin{aligned} y' &= 3 \sin^2(x^3 + \ln x) [\sin(x^3 + \ln x)]' \\ &= 3 \sin^2(x^3 + \ln x) \cos(x^3 + \ln x) (x^3 + \ln x)' \\ &= 3 \sin^2(x^3 + \ln x) \cos(x^3 + \ln x) \left(3x^2 + \frac{1}{x} \right) \end{aligned}$$

Issues:

- Should you write $\sin^n(\cdot)$ or write $[\sin(\cdot)]^n$.
 - Did the students know chain rule or is this the first time they are seeing it— how many chain rule problems have they done?
 - Do students really understand the algebra—composition of functions and the fact that sin is a function (do the students write $\sin x = x \sin$, not to mention \sin^2).
 - How many steps should you skip and which ones?
 - Should you write the chain rule on the board? If you do which notation do you write? ($f(g(x))$ or $\frac{dy}{du} \frac{du}{dx}$)
 - Should you define $f(x)$ and $g(x)$? If you define f and g , should you call it $f(u)$ or $f(x)$?
 - What is a correct answer? How simplified should the answer be?
2. Sketch the graph of $y = x^4 - 4x^3 + 10$.
Solution: $y' = 4x^3 - 12x^2$, $y'' = 12x^2 - 24x$. Critical numbers are $x = 0, 3$. Possible inflection points are $x = 0, 2, 3$. Test all these points and get the graph:



3. Using L'Hospital's rule, find the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x + x^2}$$

4. Find the interval $\int_{-2}^3 x^3 dx$

5. Find the limits

$$\lim_{x \rightarrow 6} \frac{x^2 - 36}{x - 6} =$$
$$\lim_{y \rightarrow 8} \frac{(y - 8)^2}{y^3 - 6y^2 - 17y + 8} =$$
$$\lim_{t \rightarrow 0} \frac{\sin^2(4t)}{t} =$$

6. Find the derivatives

$$\frac{d}{dx} \int_{x^7}^1 \sin^2 t \, dt =$$
$$\frac{d}{dx} \int_0^x (6t^3 - 3t) \, dt =$$
$$\frac{d}{dw} \int_{-8}^{\cos w} 4u^4 - 6u \, du =$$