# 2.2 Changing One Dimension

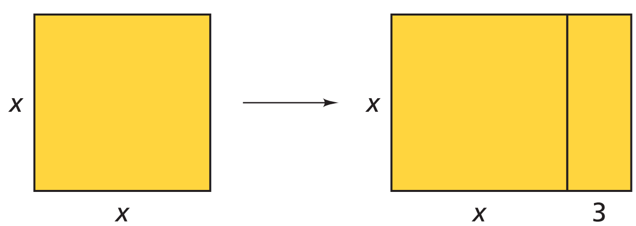
The expression $(n - 2)(n + 2)$ is in **factored form** because it is written as a product of factors. The expression $n^2 - 4$ is in **expanded form** because it is written as the sum or difference of terms. A **term** is an expression that consists of variables and/or numbers multiplied together. Specifically, $n^2 - 4$ is the difference of the terms $n^2$ and 4.

The expressions $(n - 2)(n + 2)$ and $n^2 - 4$ are *equivalent*. This means $(n - 2)(n + 2) = n^2 - 4$ is true for every value of $n$.

## Getting Ready for Problem 2.2

A square has sides of lengths $x$ centimeters. One dimension of the square is increased by 3 centimeters to create a new rectangle.

- How do the areas of the square and the new rectangle compare?



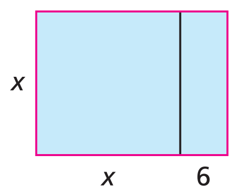

- Write two expressions for the area of the new rectangle. How do you know that the expressions are equivalent?

The expressions $x(x + 3)$ and $x^2 + 3x$ are examples of quadratic expressions. An expression in factored form is quadratic if it has exactly two linear factors, each with the variable raised to the first power. An expression in expanded form is quadratic if the highest power of the variable is 2.

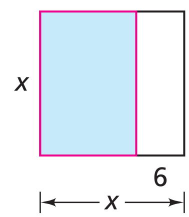
## Problem 2.2 Quadratic Expressions

**A.** Each diagram shows a large rectangle divided into two smaller rectangles. Write two expressions, one in factored form and one in expanded form, for the area of the rectangle outlined in red.

**1.**




**2.**




**B.** Complete the steps in bullets for each of the factored expressions in parts (1)–(3).

- Draw a divided rectangle whose area is represented by the expression. Label the lengths and area of each section.
- Write an equivalent expression in expanded form.

**1.** $x(x + 4)$ **2.** $x(x - 4)$ **3.** $x(5 + 2)$

**C.** Complete the steps in bullets for each of the factored expressions in parts (1)–(3).

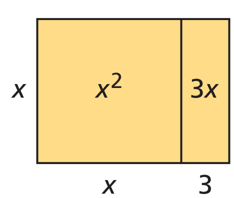
- Draw a divided rectangle whose area is represented by the expression. Label the lengths and area of each section.
- Tell what clues in the expanded expression helped you draw the divided rectangle.
- Write an equivalent expression in factored form.

**1.** $x^2 + 5x$ **2.** $x^2 - 5x$ **3.** $5x + 4x$

ACE **Homework starts on page 30.**

## 2.3 Changing Both Dimensions

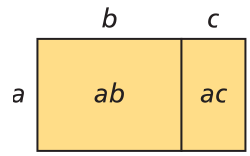
You can write the area of the larger rectangle below as $x(x + 3)$ or $x^2 + 3x$.



The equation $x(x + 3) = x^2 + 3x$ is an example of the **Distributive Property,** which you studied in earlier units.

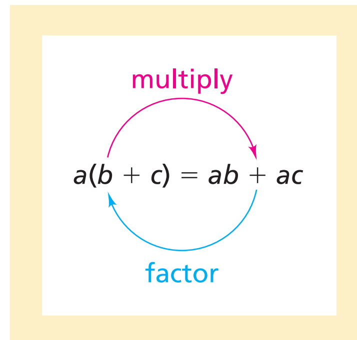
The Distributive Property says that, for any three numbers *a, b,* and *c,*

$a(b + c) = ab + bc.$




Area:
$a(b + c)$ or $ab + ac$

When you write $a(b + c)$ as $ab + ac$, you are *multiplying*, or writing the expression in expanded form. When you write $ab + ac$ as $a(b + c)$, you are *factoring*, or writing the expression in factored form.

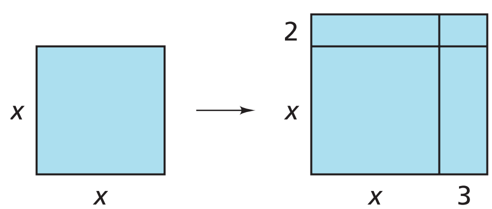



The terms $2x$ and $3x$ are *like terms*. The Distributive Property can be used to add like terms. For example, $2x + 3x = (2 + 3)x = 5x$.

In Problem 2.3, you will explore what happens to the area of a square when both dimensions are changed. You will see how the Distributive Property can be used to change the expression for area from factored form to expanded form.

## Getting Ready for Problem 2.3

A new rectangle is made by increasing one side of a square with sides of length $x$ by 2 centimeters and increasing the other side by 3 centimeters.

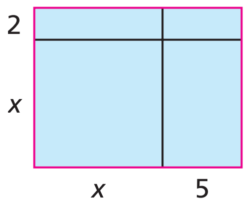



- How do the areas of the square and the new rectangle compare?
- How can you represent the area of the new rectangle?

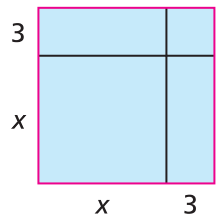
## Problem 2.3 The Distributive Property

**A.** Each rectangle has been subdivided into four smaller rectangles. Write two expressions for the area of the rectangle outlined in red, one in factored form and one in expanded form.

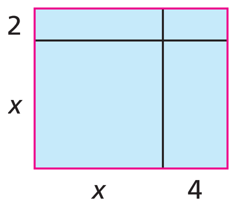
**1.**



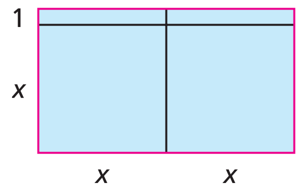

**2.**




**3.**




**4.**




**B.** A square has sides of length $x$ centimeters. One dimension is doubled and then increased by 2 centimeters. The other dimension is increased by 3 centimeters.

**1.** Make a sketch of the new rectangle. Label the area of each section.

**2.** Write two expressions, one in factored form and one in expanded form, for the area of the new rectangle.

**C.** Use a rectangle model to help write each expression in expanded form.

**1.** $(x + 3)(x + 5)$ **2.** $(4 + x)(4 + x)$ **3.** $3x(x + 1)$

**D.** Carminda says she doesn't need a rectangle model to multiply $(x + 3)$ by $(x + 2)$. She uses the Distributive Property.

$$(x + 3)(x + 2) = (x + 3)x + (x + 3)2 \qquad (1)$$
$$= x^2 + 3x + 2x + 6 \qquad (2)$$
$$= x^2 + 5x + 6 \qquad (3)$$

**1.** Is Carminda correct? Explain what she did at each step.

**2.** Show how using the Distributive Property to multiply $(x + 3)$ and $(x + 5)$ is the same as using a rectangle model.

**E.** Use the Distributive Property to write each expression in expanded form.

**1.** $(x + 5)(x + 5)$ **2.** $(x + 3)(x - 4)$ **3.** $2x(5 - x)$

**4.** $(x - 3)(x - 4)$ **5.** $(x + 2)(x - 2)$

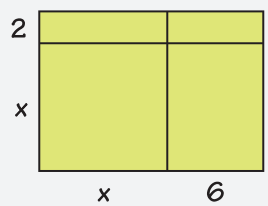
ACE **Homework starts on page 30.**

## 2.4 Factoring Quadratic Expressions

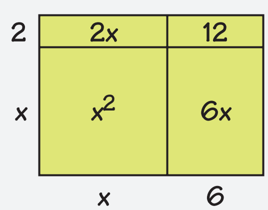
You know two ways to change a factored expression, such as $(x + 2)(x + 6)$, to expanded form.

**Rectangle Model**

Subdivide.




Label areas.




Add the areas of the sections:

$(x + 2)(x + 6) = x^2 + 2x + 6x + 12$
$= x^2 + 8x + 12$

**Distributive Property**

$(x + 2)(x + 6) = (x + 2)x + (x + 2)6$

$= x^2 + 2x + 6x + 12$

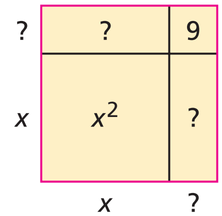
$= x^2 + 8x + 12$

*How can you write an expanded expression, such as $x^2 + 8x + 12$, in factored form?*

In the next problem, we will use the distributive property to write expressions in factored form.

## Problem 2.4 Factoring Quadratic Expressions

**A. 1.** Copy the diagram. Replace each question mark with the correct length or area.




**2.** Write two expressions for the area of the rectangle outlined in red.

**B.** Consider this expression.

$$x^2 + bx + 8$$

**1.** Choose a value for $b$ that gives an expression you can factor. Then, write the expression in factored form.

**2.** Compare your work with your classmates. Did everyone write the same expressions? Explain.

**C.** In parts (1)–(4), find values of $r$ and $s$ that make the equations true.

**1.** $x^2 + 10x + 24 = (x + 6)(x + r)$

**2.** $x^2 + 11x + 24 = (x + s)(x + r)$

**3.** $x^2 + 25x + 24 = (x + r)(x + s)$

**4.** Describe the strategies you used to factor the expressions in parts (1)–(3).

**D.** Alyse says she can use the Distributive Property to factor the expression $x^2 + 10x + 16$. She writes:

$$x^2 + 10x + 16 = x^2 + 2x + 8x + 16 \quad (1)$$
$$= x(x + 2) + 8(x + 2) \quad (2)$$
$$= (x + 2)(x + 8) \quad (3)$$

Is Alyse correct? Explain what she did at each step.

**E.** Use the Distributive Property to factor each expression.

**1.** $x^2 + 5x + 2x + 10$ **2.** $x^2 + 11x + 10$ **3.** $x^2 + 3x - 10$

**4.** $x^2 + 16x + 15$ **5.** $x^2 - 8x + 15$ **6.** $x^2 - 12x + 36$

ACE **Homework starts on page 30.**

Applications

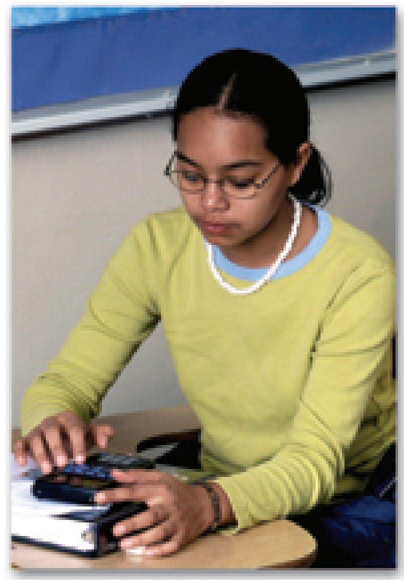
Connections

Extensions

ACE

## Applications

**1.** A square has sides of length $x$ centimeters. One dimension increases by 4 centimeters and the other decreases by 4 centimeters, forming a new rectangle.



**a.** Make a table showing the side length and area of the square and the area of the new rectangle. Include whole-number $x$-values from 4 to 16.

**b.** On the same axes, graph the ($x$, *area*) data for both the square and the rectangle.

**c.** Suppose you want to compare the area of a square with the area of the corresponding new rectangle. Is it easier to use the table or the graph?

**d.** Write equations for the area of the original square and the area of the new rectangle in terms of $x$.

**e.** Use your calculator to graph both equations. Show values of $x$ from $-10$ to 10. Copy the graphs onto your paper. Describe the relationship between the two graphs.

**2.** A square has sides of length $x$ centimeters. One dimension increases by 5 centimeters, forming a new rectangle.

**a.** Make a sketch to show the new rectangle.

**b.** Write two equations, one in factored form and one in expanded form, for the area of the new rectangle.

**c.** Graph the equations in part (b).

**For Exercises 3 and 4, draw a divided rectangle whose area is represented by each expression. Label the lengths and area of each section. Then, write an equivalent expression in expanded form.**

**3.** $x(x + 7)$

**4.** $x(x - 3)$

For Exercises 5–7, draw a divided rectangle whose area is represented by each expression. Label the lengths and area of each section. Then, write an equivalent expression in factored form.

**5.** $x^2 + 6x$ **6.** $x^2 - 8x$ **7.** $x^2 - x$

For Exercises 8–11, write the expression in factored form.

**8.** $x^2 + 10x$ **9.** $x^2 - 6x$

**10.** $x^2 + 11x$ **11.** $x^2 - 2x$

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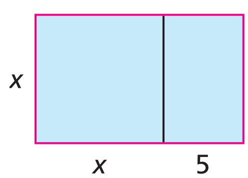
For Exercises 12–15, write the expression in expanded form.

**12.** $x(x + 1)$ **13.** $x(x - 10)$

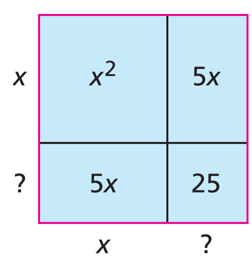
**14.** $x(x + 6)$ **15.** $x(x - 15)$

For Exercises 16–20, write two expressions, one in factored form and one in expanded form, for the area of the rectangle outlined in red.

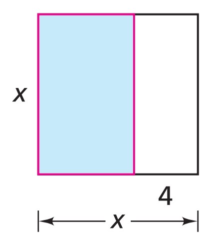
**16.**



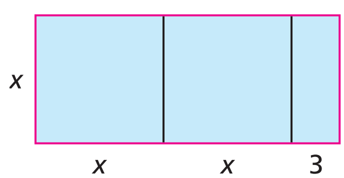

**17.**



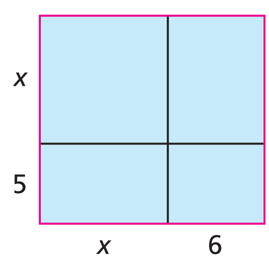

**18.**




**19.**




**20.**




**21.** A square has sides of length $x$ meters. Both dimensions increase by 5 meters, forming a new square.

**a.** Make a sketch to show the new square.

**b.** Write two equations, one in factored form and one in expanded form, for the area of the new square in terms of $x$.

**c.** Graph the equations in part (b).

Applications

**22.** A square has sides of length $x$ centimeters. One dimension increases by 4 centimeters and the other increases by 5 centimeters, forming a new rectangle.

**a.** Make a sketch to show the new rectangle.

**b.** Write two equations, one in factored form and one in expanded form, for the area of the new rectangle.

**c.** Graph the equations in part (b).

**For Exercises 23–28, use the Distributive Property to write each expression in expanded form.**

**23.** $(x - 3)(x + 4)$ **24.** $(x + 3)(x + 5)$ **25.** $x(x + 5)$

**26.** $(x - 2)(x - 6)$ **27.** $(x - 3)(x + 3)$ **28.** $(x - 3)(x + 5)$

**29. a.** Draw and label a rectangle whose area is represented by each expression.

$x^2 + 3x + 4x + 12$ $x^2 + 7x + 10$

**b.** For each expression in part (a), write an equivalent expression in factored form.

**30.** Write each expression in factored form.

**a.** $x^2 + 13x + 12$ **b.** $x^2 - 13x + 12$ **c.** $x^2 + 8x + 12$

**d.** $x^2 - 8x + 12$ **e.** $x^2 + 7x + 12$ **f.** $x^2 - 7x + 12$

**g.** $x^2 + 11x - 12$ **h.** $x^2 - 11x - 12$ **i.** $x^2 + 4x - 12$

**j.** $x^2 - 4x - 12$ **k.** $x^2 + x - 12$ **l.** $x^2 - x - 12$

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**For Exercises 31–39, determine whether the equation represents a quadratic relationship *without* making a table or a graph. Explain.**

**31.** $y = 5x + x^2$ **32.** $y = 2x + 8$ **33.** $y = (9 - x)x$

**34.** $y = 4x(3 + x)$ **35.** $y = 3^x$ **36.** $y = x^2 + 10x$

**37.** $y = x(x + 4)$ **38.** $y = 2(x + 4)$ **39.** $y = 7x + 10 + x^2$

Applications

40. Give the line of symmetry, the $x$- and $y$-intercepts, and the maximum or minimum point for the graph of each equation.

a. $y = (x - 3)(x + 3)$
b. $y = x(x + 5)$
c. $y = (x + 3)(x + 5)$
d. $y = (x - 3)(x + 5)$
e. $y = (x + 3)(x - 5)$

For Exercises 41 and 42, complete parts (a)–(e) for each equation.

41. $y = x^2 + 5x + 6$
42. $y = x^2 - 25$

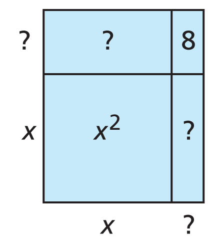
a. Find an equivalent factored form of the equation.
b. Identify the $x$- and $y$-intercepts for the graph of the equation.
c. Find the coordinates of the maximum or minimum point.
d. Find the line of symmetry.
e. Tell which form of the equation can be used to predict the features in parts (b)–(d) without making a graph.

43. Darnell makes a rectangle from a square by doubling one dimension and adding 3 centimeters. He leaves the other dimension unchanged.

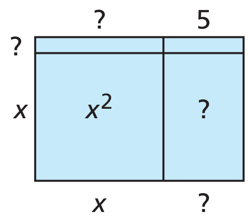
a. Write an equation for the area $A$ of the new rectangle in terms of the side length $x$ of the original square.
b. Graph your area equation.
c. What are the $x$-intercepts of the graph? How can you find the $x$-intercepts from the graph? How can you find them from the equation?

# Connections

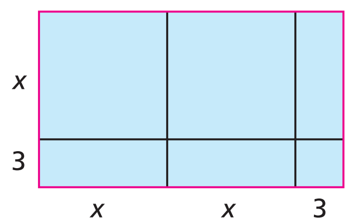
**20.** **a.** Make sketches that show two ways of completing the rectangle model at the right using whole numbers. For each sketch, express the area of the largest rectangle in both expanded form and factored form.




**b.** Is there more than one way to complete the rectangle model below using whole numbers? Explain.




**21.** Write two equivalent expressions for the area of the rectangle outlined in red below.




**22.** Consider these quadratic expressions.

$$2x^2 + 7x + 6 \qquad\qquad x^2 + 6x + 8$$

**a.** For each expression, sketch a rectangle whose area the expression represents.

**b.** Write each expression in factored form. Which expression is easier to factor? Why?

**For Exercises 23–28, write the expression in expanded form.**

| | | |
|---|---|---|
| **29** $x(5 - x)$ | **24.** $(x + 1)(x + 3)$ | **25.** $(x - 1)(x + 3)$ |
| **26.** $3x(x + 5)$ | **27.** $(2x + 1)(x + 3)$ | **28.** $(2x - 1)(x + 3)$ |

**For Exercises 29–35, write the expression in factored form.**

| | | |
|---|---|---|
| **29.** $x^2 - 9x + 8$ | **30.** $4x^2 - 6x$ | **31.** $x^2 - 2x - 3$ |
| **32.** $3x^2 + 14x + 8$ | **33.** $4x^2 + 6x$ | **34.** $4x^2 - x - 3$ |
| **35.** $x^3 - 2x^2 - 3x$ | | |

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