Investigation 1

Equivalent Expressions

When you want to communicate an idea in words, you can usually express it in many ways. All the statements below communicate the same information about Mika and Jim.

- Jim is older than Mika.
- Mika is younger than Jim.
- Jim was born before Mika.
- Mika was born after Jim.

Can you think of other ways to express the same idea?

Symbolic expressions, formulas, and equations are valuable tools in mathematics. The formula $P = 2L + 2W$ gives directions for calculating the perimeter of any rectangle with length L and width W.

Since you can usually think about a situation in more than one way, you can often express the situation in symbols in more than one way.

Getting Ready for Problem 1.1

Jim says the perimeter of the rectangle above is $P = 2(L + W)$. Mika says the perimeter is $P = 2L + 2W$.

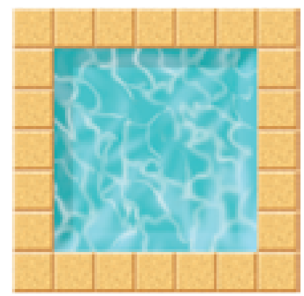
- Why do you think Jim used parentheses in his equation?
- Are the expressions $2L + 2W$ and $2(L + W)$ *equivalent*? Do they produce the same perimeter for any given pair of lengths and widths? Explain your reasoning.

Since $2(L + W)$ and $2L + 2W$ represent the same quantity (the perimeter of a rectangle), they are **equivalent expressions.** This investigation explores situations in which a quantity is described with several different, but equivalent, expressions. The question is:

How can we determine if two expressions are equivalent?

1.1 Tiling Pools

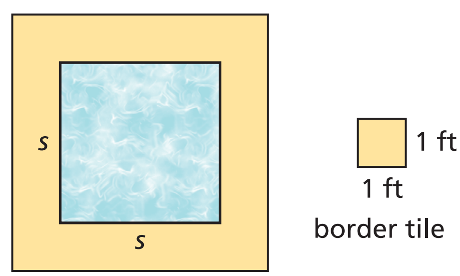
In-ground pools are often surrounded by borders of tiles. The Custom Pool Company gets orders for square pools of different sizes. For example, the pool at the right has side lengths of 5 feet and is surrounded by square border tiles. All Custom Pool border tiles measure 1 foot on each side.



- How many border tiles do you need to surround a square pool?

Problem 1.1 Writing Equivalent Expressions

In order to calculate the number of tiles needed for a project, the Custom Pool manager wants an equation relating the number of border tiles to the size of the pool.



A. **1.** Write an expression for the number of border tiles N based on the side length s of a square pool.

2. Write a different but equivalent expression for the number of tiles N needed to surround such a square pool.

3. Explain why your two expressions for the number of border tiles are equivalent.

B. **1.** Use each expression in Question A to write an equation for the number of border tiles N. Make a table and a graph for each equation.

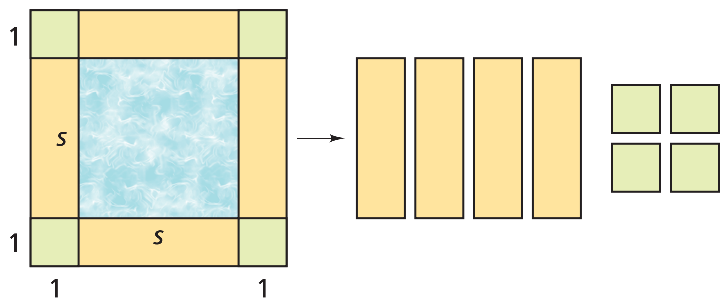
2. Based on your table and graph, are the two expressions for the number of border tiles in Question A equivalent? Explain.

C. Is the relationship between the side length of the pool and the number of border tiles linear, exponential, quadratic, or none of these? Explain.

ACE **Homework starts on page 12.**

1.2 Thinking in Different Ways

When Takashi reported his ideas about an equation relating N and s in Problem 1.1, he made the following sketch.



- What equation do you think Takashi wrote to relate N and s?

Problem 1.2 Determining Equivalence

A. Four students in Takashi's class came up with different equations for counting the number of border tiles. For each equation, make a sketch that shows how the student might have been thinking about the border of the pool.

1. Stella's equation: $N = 4(s + 1)$
2. Jeri's equation: $N = s + s + s + s + 4$
3. Hank's equation: $N = 4(s + 2)$
4. Sal's equation: $N = 2s + 2(s + 2)$

B. Use each equation in Question A to find the number of border tiles needed for a square pool with a side length of 10 feet. Can you conclude from your results that all the expressions for the number of tiles are equivalent? Explain your reasoning.

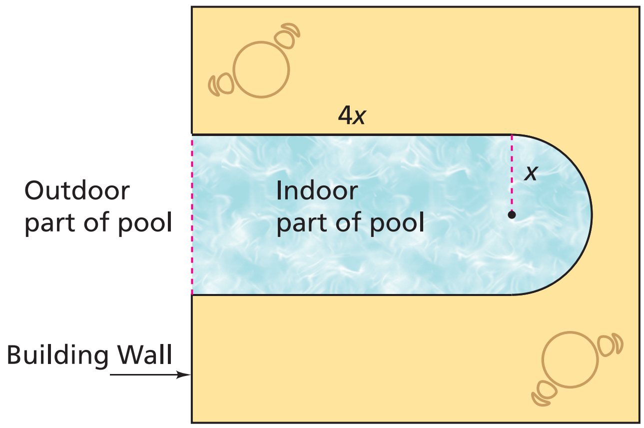
C. Which of the expressions for the number of border tiles in Question A are equivalent to Takashi's expression? Explain.

ACE **Homework starts on page 12.**

1.3 The Community Pool Problem

In this problem, we will interpret symbolic statements and use them to make predictions.

A community center is building a pool, part indoor and part outdoor. A diagram of the indoor part of the pool is shown. The indoor shape is made from a half-circle with radius x and a rectangle with length $4x$.



Problem 1.3 Interpreting Expressions

The exact dimensions of the community center pool are not available, but the area A of the whole pool is given by the equation:

$$A = x^2 + \frac{\pi x^2}{2} + 8x^2 + \frac{\pi x^2}{4}$$

A. Which part of the expression for area represents

1. the area of the indoor part of the pool? Explain.

2. the area of the outdoor part of the pool? Explain.

B. **1.** Make a sketch of the outdoor part. Label the dimensions.

2. If possible, draw another shape for the outdoor part of the pool. If not, explain why not.

C. Stella and Jeri each rewrote the expression for the area of the outdoor part of the pool to help them make a sketch.

$$\text{Stella: } x^2 + \frac{\pi x^2}{8} + \frac{\pi x^2}{8}$$

$$\text{Jeri: } \left(\frac{1}{2}x\right)\left(2x\right) + \frac{\pi x^2}{4}$$

1. Explain the reasoning each person may have used to write their expression.
2. Decide if these expressions are equivalent to the original expression in Question A, part (2). Explain your reasoning.

D. Does the equation for the area of the pool represent a linear, exponential, or quadratic relationship, or none of these? Explain.

ACE **Homework starts on page 12.**

1.4 Diving In

In the pool tile problems, you found patterns that could be represented by several different but equivalent symbolic expressions, such as:

$$4s + 4$$
$$4(s + 1)$$
$$s + s + s + s + 4$$
$$2s + 2(s + 2)$$

The equivalence of these expressions can be shown with arrangements of tiles. Equivalence also follows from properties of numbers and operations.

An important property is the **Distributive Property:**

For any real numbers a, b, and c:

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

For example, this property guarantees that $4(s + 1) = 4s + 4$ for any s.

We say that $a(b + c)$ and $4(s + 1)$ are in *factored form* and $ab + ac$ and $4s + 4$ are in *expanded form*.

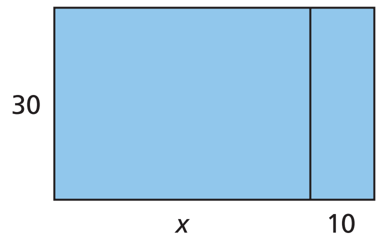
The next problem reviews the Distributive Property.

Getting Ready for Problem 1.4

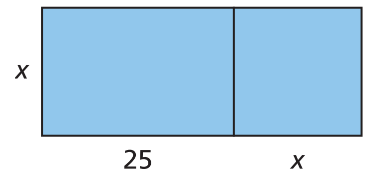
Swimming pools are sometimes divided into sections that are used for different purposes. A pool may have a section for swimming laps and a section for diving, or a section for experienced swimmers and a section for small children.

Below are diagrams of pools with swimming and diving sections. The dimensions are in meters.

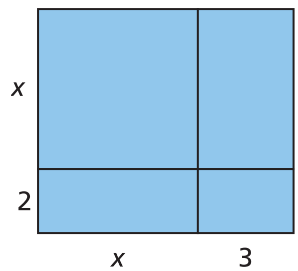
1.



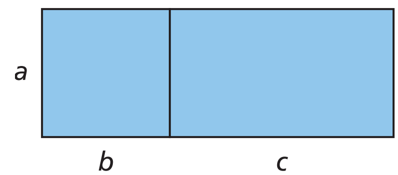
2.



3.



4.



- For each pool, write two different but equivalent expressions for the total area.
- Explain how these diagrams and expressions illustrate the Distributive Property.

The Distributive Property, as well as the Commutative Property and other properties for numbers, are useful for writing equivalent expressions. The Commutative Property states that $a + b = b + a$ and $ab = ba$, where a and b are real numbers. These properties were discussed in previous units.

Problem 1.4 Revisiting the Distributive Property

A. Write each expression in expanded form.

1. $3(x + 5)$ **2.** $2(3x - 10)$

3. $2x(x + 5)$ **4.** $(x + 2)(x + 5)$

B. Write each expression in factored form.

1. $12 + 24x$ **2.** $x + x + x + 6$

3. $x^2 + 3x$ **4.** $x^2 + 4x + 3$

C. The following expressions all represent the number of border tiles N for a square pool with side length s.

$$4(s + 1)$$
$$s + s + s + s + 4$$
$$2s + 2(s + 2)$$
$$4(s + 2) - 4$$
$$(s + 2)^2 - s^2$$

Use the Distributive and Commutative properties to show that these expressions are equivalent.

D. Three of the following expressions are equivalent. Explain which expression is not equivalent to the other three.

1. $2x - 12x + 10$ **2.** $12x - 2x + 10$

3. $10 - 10x$ **4.** $10(1 - x)$

E. Copy each equation. Insert one set of parentheses in the expression to the left of the equal sign so that it is equivalent to the expression to the right of the equal sign.

1. $6p + 2 - 2p = 4p + 12$

2. $6p + 2 - 2p = 6p$

ACE **Homework starts on page 12.**