# Expected Value and the Game of Craps 

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Craps is a gambling game found in most casinos based on rolling two six sided dice. Most players who walk into a casino and try to play craps for the first time are overwhelmed by all the possible bets. The goal here is to understand what these bets are and how the casino makes money.

## 1 Probabilities and Expected Values

Expected value is the expected return. We want to know what sort of payoff you can expect when you place a bet.

### 1.1 Some Simple Games

Lets say you play a game where you roll a fair die (what does this mean?) and get paid according to your roll:

| Roll | Payout |
| :---: | :---: |
| 6 | $\$ 4$ |
| 5 | $\$ 2$ |
| 4 | $\$ 1$ |
| 3 | $\$ 0$ |
| 2 | $\$ 0$ |
| 1 | $\$ 0$ |

You have to pay $\$ 1$ to play this game. Is it worth it? What do you expect to happen in the long run?

Here is how you might answer this:
You roll a $6, \frac{1}{6}$ of the time, and you get paid $\$ 4$.
You roll a $5, \frac{1}{6}$ of the time, and you get paid $\$ 2$.
You roll a $4, \frac{1}{6}$ of the time, and you get paid $\$ 1$.
You roll a $3, \frac{1}{6}$ of the time, and you get paid $\$ 0$.
You roll a $2, \frac{1}{6}$ of the time, and you get paid $\$ 0$.
You roll a $1, \frac{1}{6}$ of the time, and you get paid $\$ 0$.
Sum these up to find the expected value:

$$
E(X)=\left(\frac{1}{6}\right) 4+\left(\frac{1}{6}\right) 2+\left(\frac{1}{6}\right) 1+\left(\frac{1}{6}\right) 0+\left(\frac{1}{6}\right) 0+\left(\frac{1}{6}\right) 0=\frac{7}{6} \approx 1.167
$$

Thus, you expect to get $\$ 1.17$ back every time you play, making a cool $\$ 0.17$ profit.

Another way to ask this very same question would be, "how much is the fair price for this game?" (The answer is, of course, \$1.17.)

Another way to answer this question is to use the following chart

| Roll | Profit |
| :---: | :---: |
| 6 | $\$ 3$ |
| 5 | $\$ 1$ |
| 4 | $\$ 0$ |
| 3 | $\$-1$ |
| 2 | $\$-1$ |
| 1 | $\$-1$ |

Computing expected value:

$$
E(X)=\left(\frac{1}{6}\right) 3+\left(\frac{1}{6}\right) 1+\left(\frac{1}{6}\right) 0+\left(\frac{1}{6}\right)(-1)+\left(\frac{1}{6}\right)(-1)+\left(\frac{1}{6}\right)(-1)=\frac{1}{6} \approx 0.167
$$

Again, you see that you expect about a $\$ 0.17$ profit.

### 1.1.1 Notational notes

In the first computation, we were interested in the amount of money we would get back in a single game. Thus, in this case, $X$ was this amount of money and $E(X)$ is the expected amount of money we get back.

In the second computation, we were interested in the total profit we would make. In this case, $X$ was the profit. Of course, it would have been nice to have used a different letter/variable for these things. If we did this and let $M$ be the money from one game and $P$ the profit, then we would have:

$$
P=M-1
$$

### 1.2 Some Exercises for You

Determine the expected value for the games.

1. Charge $\$ 1$ to play. Roll one die, with payouts as follows:

| Roll | Payout |
| :---: | :---: |
| 6 | $\$ 2$ |
| 5 | $\$ 2$ |
| 4 | $\$ 1$ |
| 3 | $\$ 0$ |
| 2 | $\$ 0$ |
| 1 | $\$ 1.50$ |

2. Charge: $\$ 1$ to toss 3 coins. Toss the coins. If you get all heads or all tails, you receive $\$ 5$. If not, you get nothing.
3. Charge: $\$ 1$. Roll 2 dice. If you roll 2 odd numbers, like a 3 and a 5 , you get $\$ 2$. If you roll 2 even numbers, like 4 and 6 , you get $\$ 2$. Otherwise, you get nothing.
4. Charge: $\$ 5$. Draw twice from a bag that has one $\$ 10$ and $4 \$ 1$ bills. You get to keep the bills.

## 2 Probabilities Versus Odds

Lets explore this with a roll of a dice. If you roll a dice 600 times, you would expect to see the number one, 100 times:

$$
P(\text { Roll a } 1)=\frac{(\text { Chances For })}{(\text { Total Chances })}=\frac{100}{600}=\frac{1}{6}
$$

Odds on the other hand are given as:

$$
\text { Odds }(\text { Roll a } 1)=(\text { Chances For }):(\text { Total Chances })=100: 500=1: 5
$$

Odds are usually written this way (with a colon).

### 2.1 Exercises

If given odds, compute the probability. If given a probability, compute the odds.

1. Odds of an event are $1: 4$. What is the probability?
2. Odds of an event are $2: 5$. What is the probability?
3. Odds of an event are $3: 2$. What is the probability?
4. Odds of an event are $10: 3$. What is the probability?

5 . Odds of an event are $3: 10$. What is the probability?
6. Probability of an event is $\frac{1}{3}$. What are the odds?
7. Probability of an event is $\frac{3}{10}$. What are the odds?
8. Probability of an event is $\frac{4}{3}$. What are the odds?
9. Probability of an event is $\frac{4}{17}$. What are the odds?
10. Probability of an event is $13 \%$. What are the odds?
11. Probability of an event is $\frac{7}{99}$. What are the odds?

## 3 Probability of the dice

When throwing two dice and summing the numbers, the possible outcomes are 2 through 12 . To determine the probability of getting a number you make the observation that there are 36 different ways the two dice can be rolled.

Question 1. Compute the probabilities for the sum of two rolled dice.

## Solution:

To determine the probability of rolling a number you count the number of ways to roll that number and divide by 36 .

| Sum | Combinations | Probability |
| :---: | :---: | :---: |
| 2 | $1-1$ | $\frac{2}{36}$ |
| 3 | $1-2,2-1$ | $\frac{1}{36}=\frac{1}{18}$ |
| 4 | $1-3,2-2,3-1$ | $\frac{3}{36}=\frac{1}{12}$ |
| 5 | $1-4,2-3,3-2,4-1$ | $\frac{4}{36}=\frac{1}{9}$ |
| 6 | $1-5,2-4,3-3,4-2,5-1$ | $\frac{5}{36}$ |
| 7 | $1-6,2-5,3-4,4-3,5-2,6-1$ | $\frac{6}{36}=\frac{1}{6}$ |
| 8 | $2-6,3-5,4-4,5-3,6-2$ | $\frac{5}{36}$ |
| 9 | $3-6,4-5,5-4,6-3$ | $\frac{4}{36}=\frac{1}{9}$ |
| 10 | $4-6,5-5,6-4$ | $\frac{3}{36}=\frac{1}{12}$ |
| 11 | $5-6,6-5$ | $\frac{2}{36}=\frac{1}{18}$ |
| 12 | $6-6$ | $\frac{1}{36}$ |

## 4 Craps

In the game of craps there are a wide range of possible bets that one can make. There are single roll bets, line bets and more. The player places these bets by putting his money (gambling chips) in the appropriate place on the craps table, see Figure 1.


Figure 1: Craps Table Layout

### 4.1 Single roll bets

These bets are the easiest to understand. In a single roll bet the player is betting on a certain outcome in a single roll.

### 4.1.1 Playing the field

The most obvious single roll bet is perhaps playing the field. This bet is right in the middle of the table. On a roll of $3,4,9,10$ or 11 , the player is paid even odds and on a roll of 2 or 12 the player is paid double odds. Thus, if $\$ 1$ is bet on the field and a $3,4,9,10$ or 11 is rolled the player is paid $\$ 1$ and keeps his original $\$ 1$. If a 2 or 12 is rolled, the player is paid $\$ 2$ and keeps his original $\$ 1$.

Question 2. Compute the expected value of playing the field.
Solution: Here is the expected value of one dollar bet on the field.

$$
E(X)=2 \cdot \frac{7}{18}+3 \cdot \frac{1}{18}=\frac{17}{18} \approx 0.944
$$

In other words, in the long run $\$ 1$ bet on the field will expect to pay the player $\$ 0.944$. As we will see, this is better than some bets but it is not good enough.

### 4.1.2 C and E

These are the craps and yo bets. In the game of craps a roll of craps is a roll of a 2,3 or 12 . A roll of eleven is also called a yo. (At the craps table you will hear people calling for a "lucky-yo," meaning they want an eleven rolled.)

A player can place a one-time bet on any of these numbers and the payoffs are printed on the craps table.

Question 3. Fill in the table below.

| Roll | Odds paid | Actual Odds | Probability | Expected value of \$1 bet |
| :---: | :---: | :---: | :---: | :---: |
| 2 | $30: 1$ | $35: 1$ | $\frac{1}{36}$ | $\frac{31}{36} \approx 0.861$ |
| 3 | $15: 1$ | $17: 1$ | $\frac{1}{18}$ | $\frac{8}{9} \approx 0.889$ |
| Yo 11 | $15: 1$ | $17: 1$ | $\frac{1}{18}$ | $\frac{8}{9} \approx 0.889$ |
| 12 | $30: 1$ | $35: 1$ | $\frac{1}{36}$ | $\frac{31}{36} \approx 0.861$ |
| Any Crap | $7: 1$ | $8: 1$ | $\frac{1}{9}$ | $\frac{8}{9} \approx 0.889$ |
| Any 7 | $4: 1$ | $5: 1$ | $\frac{1}{6}$ | $\frac{5}{6} \approx 0.833$ |

Notice that the odds paid are printed on the table. So, for example, the odds paid for any seven is 4 to 1 . Thus, if you put $\$ 1$ down and a seven is rolled this will pay you $\$ 4$ plus your original bet (thus you will walk away having "earned" \$4).

Note that in this table we introduced the column "Actual Odds." This is the odds that the casino should pay in order to be completely fair. In other words, if the casino paid these odds then the expected value of a dollar bet would be a dollar.

### 4.2 The pass line

Line bets are the best example of a multi-roll bet. This means that the bet actually lasts several rolls-the player does not generally win or lose immediately.

### 4.2.1 The basic pass line bet

When betting the pass line the player puts his bet right on the pass line, say $\$ 1$. Then, the first roll is called the come-out roll. This bet is immediately won if a 7 or 11 is rolled (paid even odds) and immediately lost of a craps $(2,3,12)$ is rolled. If any other number is rolled then a point is established and rolling continues until either that point is rolled again (good for the pass-line better) or a 7 is rolled (good for the casino). Here is a typical sequence of rolls starting with the player putting $\$ 1$ on the pass line.

| Roll | Description |
| :---: | :--- |
| 3 | Come out roll, craps. Player loses $\$ 1$ and places another $\$ 1$ on pass line. |
| 7 | Come out roll, lucky 7 . Player is paid $\$ 1$ and keep his $\$ 1$ on the pass line. |
| 11 | Come out roll, Yo 11. Player is paid $\$ 1$ and keep his $\$ 1$ on the pass line. |
| 4 | Come out roll, point of 4 established. No payouts |
| 6 | Nothing happens, point is 4 . |
| 9 | Nothing happens, point is 4. |
| 12 | Nothing happens, point is 4. |
| 4 | Point is made. Player is paid $\$ 1$ and keeps his $\$ 1$ on the pass line. |
| 6 | Come out roll, point of 6 is established. |
| 3 | Nothing happens, point is 6 . |
| 7 | 7 -out. Player loses $\$ 1$ bet. |

Once a point has been established the player wants that point to be rolled and loses if a 7 is rolled. Just looking at probabilities, it is clear that the player is more likely to win if the point is 6 or 8 and less likely to win if the point is 4 or 10 .

Lets compute some expected values. The key in computing the probabilities involved is first understanding that a pass line bet begins on the come out roll and ends when either the player is paid or when the player loses. Theoretically, this could take forever (nothing guarantees that the point or a 7 will ever be rolled). In any case, for the come-out roll, we can say the following:

$$
\begin{aligned}
P(7 \text { or } 11) & =\frac{2}{9} & & \text { (Player wins) } \\
P(2,3 \text { or } 12) & =\frac{1}{9} & & \text { (Player craps out, loses) } \\
P(4,5,6,8,9,10) & =\frac{2}{3} & & \text { (A point is established) }
\end{aligned}
$$

Once a point is established, we want to compute the probability of that point coming up again before a seven. This will, of course, depend on what that point is.

Lets do the computation for the point 4 (the calculation if the point was 10 is identical). The player will win if a 4 rolls before a seven. So, a winning sequence of rolls could look like

4 player rolls a 4 right away
or
$3,6,5,8,4$ player rolls something other than a 4 or 7 and then finally a 4
So, the following are the probabilities of winning after a point of 4 has been established.

$$
\begin{aligned}
P(4 \text { on roll } 1) & =\frac{1}{12} \\
P(4 \text { on roll } 2, \text { no } 7) & =P(\text { not a } 4 \text { or } 7) P(4)=\frac{3}{4} \cdot \frac{1}{12} \\
P(4 \text { on roll } 3) & =P(\text { not a } 4 \text { or } 7) P(4 \text { on roll } 2, \text { no } 7)=\left(\frac{3}{4}\right)^{2} \cdot \frac{1}{12}
\end{aligned}
$$

Putting all this together, the probability of winning after a point of 4 has been established is a geometric series:
$P(4$ before 7$)=P(4$ on roll 1$)+P(4$ on roll 2 , no 4 or 7 before roll 2$)+\cdots$

$$
\begin{aligned}
& =\frac{1}{12}+\left(\frac{3}{4}\right) \cdot \frac{1}{12}+\left(\frac{3}{4}\right)^{2} \cdot \frac{1}{12}+\left(\frac{3}{4}\right)^{3} \cdot \frac{1}{12}+\cdots \\
& =\frac{1}{12} \sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^{n}=\frac{1}{12}\left(\frac{1}{1-\frac{3}{4}}\right)=\frac{1}{3}
\end{aligned}
$$

Similarly, we can compute the probability of winning other points. (And, included are the "points" of 2,3 and 12 , which are not really points because they are craps.)

$$
\begin{aligned}
& P(2 \text { before } 7)=P(12 \text { before } 7)=\frac{1}{7} \\
& P(3 \text { before } 7)=P(11 \text { before } 7)=\frac{1}{4} \\
& P(4 \text { before } 7)=P(10 \text { before } 7)=\frac{1}{3} \\
& P(5 \text { before } 7)=P(9 \text { before } 7)=\frac{2}{5} \\
& P(6 \text { before } 7)=P(8 \text { before } 7)=\frac{5}{11}
\end{aligned}
$$

We can now compute our probabilities of winning

| Event | Probability |
| :---: | ---: |
| Win with a 7 or 11 on come-out roll | $\frac{2}{9}$ |
| Lose with 2,3,12 on come-out roll | $\frac{1}{9}$ |
| Establish point 4 and win | $\frac{1}{12}\left(\frac{1}{3}\right)=\frac{1}{36}$ |
| Establish point 4 and lose | $\frac{1}{12}\left(\frac{2}{3}\right)=\frac{1}{18}$ |
| Establish point 5 and win | $\frac{1}{9}\left(\frac{2}{5}\right)=\frac{2}{45}$ |
| Establish point 5 and lose | $\frac{1}{9}\left(\frac{3}{5}\right)=\frac{1}{15}$ |
| Establish point 6 and win | $\frac{5}{36}\left(\frac{5}{11}\right)=\frac{25}{396}$ |
| Establish point 6 and lose | $\frac{5}{36}\left(\frac{6}{11}\right)=\frac{5}{66}$ |
| Establish point 8 and win | $\frac{5}{36}\left(\frac{5}{11}\right)=\frac{25}{396}$ |
| Establish point 8 and lose | $\frac{5}{36}\left(\frac{6}{11}\right)=\frac{5}{66}$ |
| Establish point 9 and win | $\frac{1}{9}\left(\frac{2}{5}\right)=\frac{2}{45}$ |
| Establish point 9 and lose | $\frac{1}{9}\left(\frac{3}{5}\right)=\frac{1}{15}$ |
| Establish point 10 and win | $\frac{1}{12}\left(\frac{1}{3}\right)=\frac{1}{36}$ |
| Establish point 10 and lose | $\frac{1}{12}\left(\frac{2}{3}\right)=\frac{1}{18}$ |

Computing expected values of a $\$ 1$ bet on the pass lines gives

$$
\begin{aligned}
E(X) & =2 \cdot \frac{2}{9}+2\left(\frac{1}{36}+\frac{2}{45}+\frac{25}{396}+\frac{25}{396}+\frac{2}{45}+\frac{1}{36}\right) \\
& =\frac{488}{495} \approx 0.98586
\end{aligned}
$$

### 4.2.2 Laying odds

The pass line has one of the best expected values for your dollar in the casino. But, craps gives the player another method for increasing your odds. Once a point is established, the player is allowed to back his bet up with an odds bet, paid off at true odds. What this means is that the expected value of a $\$ 1$ odds bet is $\$ 1$.

## 5 Playing "Don't Pass"

In addition to the "Pass line" there is also the "Don't Pass line" where a player is betting with the casino. In other words, the "Don't Pass" player will win when all the other players are losing. This is true except that the "Don't Pass line" is actually the "Don't Pass bar 12" and the player does not win when on the come out roll when a craps of 12 is rolled.

So, what is the house advantage in this case? We compute expected value of $\$ 1$ placed on the "Don't Pass line." Notice that our table is essentially the same as before.

| Event | Probability |
| :---: | ---: |
| Lose with a 7 or 11 on come-out roll | $\frac{2}{9}$ |
| Win with 2,3 on come-out roll | $\frac{1}{12}$ |
| Push with 12 on come-out roll | $\frac{1}{36}$ |
| Establish point 4 and lose | $\frac{1}{12}\left(\frac{1}{3}\right)=\frac{1}{36}$ |
| Establish point 4 and win | $\frac{1}{12}\left(\frac{2}{3}\right)=\frac{1}{18}$ |
| Establish point 5 and lose | $\frac{1}{9}\left(\frac{2}{5}\right)=\frac{2}{45}$ |
| Establish point 5 and win | $\frac{1}{9}\left(\frac{3}{5}\right)=\frac{1}{15}$ |
| Establish point 6 and lose | $\frac{5}{36}\left(\frac{5}{11}\right)=\frac{25}{396}$ |
| Establish point 6 and win | $\frac{5}{36}\left(\frac{6}{11}\right)=\frac{5}{66}$ |
| Establish point 8 and lose | $\frac{5}{36}\left(\frac{5}{11}\right)=\frac{25}{396}$ |
| Establish point 8 and win | $\frac{5}{36}\left(\frac{6}{11}\right)=\frac{5}{66}$ |
| Establish point 9 and lose | $\frac{1}{9}\left(\frac{2}{5}\right)=\frac{2}{45}$ |
| Establish point 9 and win | $\frac{1}{9}\left(\frac{3}{5}\right)=\frac{1}{15}$ |
| Establish point 10 and lose | $\frac{1}{12}\left(\frac{1}{3}\right)=\frac{1}{36}$ |
| Establish point 10 and win | $\frac{1}{12}\left(\frac{2}{3}\right)=\frac{1}{18}$ |

Computing expected values of a $\$ 1$ bet on the pass lines gives

$$
\begin{aligned}
E(X) & =2 \cdot \frac{1}{12}+\frac{1}{36}+4\left(\frac{1}{18}+\frac{1}{15}+\frac{5}{66}\right) \\
& =\frac{217}{220} \approx 0.9864
\end{aligned}
$$

Which is slightly better than playing the pass line.

## 6 Crapless Craps

There is a variation of craps (discovered by the author on a recent trip to a casino) called "Crappless Craps" or "No More Craps, Craps." What this means is that the craps and yo are all done away with on the come out roll. Instead, the $2,3,11$, and 12 are valid points just like the $4,5,6,8,9$ and 10 . The odds bets for these are also paid off at true odds. This looks like a good thing because there are less ways to lose on the come-out roll.

We now compute some probabilities and expected values.

| Event | Probability |
| :---: | :---: |
| Win with 7 on come-out roll | $\frac{1}{6}$ |
| Establish point 2 and win | $\frac{1}{36}\left(\frac{1}{7}\right)=\frac{1}{252}$ |
| Establish point 2 and lose | $\frac{1}{36}\left(\frac{6}{7}\right)=\frac{1}{42}$ |
| Establish point 3 and win | $\frac{1}{18}\left(\frac{1}{4}\right)=\frac{1}{72}$ |
| Establish point 3 and lose | $\frac{1}{18}\left(\frac{3}{4}\right)=\frac{1}{24}$ |
| Establish point 4 and win | $\frac{1}{12}\left(\frac{1}{3}\right)=\frac{1}{36}$ |
| Establish point 4 and lose | $\frac{1}{12}\left(\frac{2}{3}\right)=\frac{1}{18}$ |
| Establish point 5 and win | $\frac{1}{9}\left(\frac{2}{5}\right)=\frac{2}{45}$ |
| Establish point 5 and lose | $\frac{1}{9}\left(\frac{3}{5}\right)=\frac{1}{15}$ |
| Establish point 6 and win | $\frac{5}{36}\left(\frac{5}{11}\right)=\frac{25}{396}$ |
| Establish point 6 and lose | $\frac{5}{36}\left(\frac{6}{11}\right)=\frac{5}{66}$ |
| Establish point 8 and win | $\frac{5}{36}\left(\frac{5}{11}\right)=\frac{25}{396}$ |
| Establish point 8 and lose | $\frac{5}{36}\left(\frac{6}{11}\right)=\frac{5}{66}$ |
| Establish point 9 and win | $\frac{1}{9}\left(\frac{2}{5}\right)=\frac{2}{45}$ |
| Establish point 9 and lose | $\frac{1}{9}\left(\frac{3}{5}\right)=\frac{1}{15}$ |
| Establish point 10 and win | $\frac{1}{12}\left(\frac{1}{3}\right)=\frac{1}{36}$ |
| Establish point 10 and lose | $\frac{1}{12}\left(\frac{2}{3}\right)=\frac{1}{18}$ |
| Establish point 11 and win | $\frac{1}{18}\left(\frac{1}{4}\right)=\frac{1}{72}$ |
| Establish point 11 and lose | $\frac{1}{18}\left(\frac{3}{4}\right)=\frac{1}{24}$ |
| Establish point 12 and win | $\frac{1}{36}\left(\frac{1}{7}\right)=\frac{1}{252}$ |
| Establish point 12 and lose | $\frac{1}{36}\left(\frac{6}{7}\right)=\frac{1}{42}$ |

Computing expected value of a $\$ 1$ bet in on this pass line.

$$
\begin{aligned}
E(X) & =2 \cdot \frac{1}{6}+2 \cdot 2\left(\frac{1}{252}+\frac{1}{72}+\frac{1}{36}+\frac{2}{45}+\frac{25}{396}\right) \\
& =\frac{6557}{6930} \approx 0.946176
\end{aligned}
$$

Amazingly, this makes it a worse bet than regular craps, by a substantial amount. Also, the casino will not allow a player to play "Don't" (why not?).

## 7 How fast will you lose money?

Here are some estimate to help compute this. Over an hour, we can expect about:

- 112: dice rolls (observed, info from Internet)
- 11: points to be established
- 2: craps on come out roll
- 4: 7 or 11 on the come out roll


### 7.1 Pass line player

Basically, this means that if you are betting on the pass line then every hour you will place around 17 bets. If you are making $\$ 100$ bets then this means you will have placed $\$ 100$ on the table 17 times and on you will receive $\$ 98.59$ back from these bets. In other words, in an hour you will have lost only about $\$ 23.97$.

### 7.2 Field player

Over an hour, the field player will make about 112 bets and, if playing $\$ 100$ a bet then the field player should receive $\$ 94.44$ from every bet. This means that this field player will lose about $\$ 622.72$.

### 7.3 How to minimize your losses?

There are clearly some bad bets, such as the field (and, the field is not even the worst bet). So, first thing to do is to not play the bad bets.

When playing the pass-line, the house edge is only about $1.41 \%$. If you play with odds and place as many odds behind your bet then you can expect to decrease the house advantage. The more odds you place on your pass-line bet the more you decrease the advantage that the house has.

### 7.4 Pass-line with odds

Here we assume a player places a pass-line bet of $\$ 1$ and then, when a point is established, places an odds bets behind the pass-line. Most casino's limit the amount of this odds bet and $10 \times$ is a high amount. What this means is that if your pass-line bet is $\$ 1$ then your odds bet can be at most $\$ 10$. Here are the payoffs on such a bet if the number is hit.

| Point | Bet | Payout |
| :---: | :---: | :---: |
| 4 | $1+10$ | $1+20$ |
| 5 | $1+10$ | $1+15$ |
| 6 | $1+10$ | $1+12$ |
| 8 | $1+10$ | $1+12$ |
| 9 | $1+10$ | $1+15$ |
| 10 | $1+10$ | $1+20$ |

So, if the point is 4 and that point is hit then the player would have $\$ 1$ on the pass-line and $\$ 10$ for odds. The player would be paid $\$ 1$ for his pass-line bet and $\$ 20$ for his odds bet.

How do you compute the house advantage now?

### 7.5 Methods to win?

I found these on the Internet:

- http://casinogambling.about.com/cs/craps/a/5count.htm
- http://homepage.ntlworld.com/dice-play/CrapsSystems.htm

As one Internet site said, "There are NO winning systems for Craps that work!" It is instructive to see if you can figure out what is wrong with these systems.

### 7.5.1 The Craps 5 Count Method

The 5-Count method for finding hot shooters and reducing a player's overall risk at the craps table starts with a point number (the "point numbers" are $4,5,6,8,9$, or 10) and ends with a point number. Let's take it step by step.
A new shooter has just received the dice and is on his come-out roll. If he rolls any of the point numbers on his come-out roll, that constitutes the 1 -count. If he rolls a 2,3 , 7,11 , or 12 on the come-out, the count has not started yet.
The second roll after the 1-count is automatically the 2 -count, regardless of what number is rolled, and this applies to the 3 -count and 4 -count as well.

The next roll after the 4 -count is the 5 -Count only if the number rolled is again one of the point numbers above. If not, it is the 4 -count and holding until the 5 -Count is achieved. When the 5 -Count is achieved, you then begin to place money at risk. You can make a come bet or make a place bet.

If you usually make three come bets you can actually make your first come bet after the three count. You would then make your second come bet after the four count. You don't take odds on your come bets until after the five count. At this time you would also make your third come bet.

### 7.5.2 The Martingale or Rothstein System

Bet 1 unit on the Pass Line. If you lose bet 3 units (double your bet plus 1). If you lose again bet 7 units (double your bet plus 1) and so on. Sooner or later you must win and you will be up. If you win then start betting again from the 1 unit up.

### 7.5.3 The Patience or Watching System

Wait until the dice pass four times and then put $\$ 4$ on them losing on the fifth roll. If you lose, put double your bet (\$8) on them losing on the next roll and so on until you win.

### 7.5.4 The Hot and Cold System

This system says you will win when the dice are hot and win when the dice are cold. You bet on the dice to win when the shooter makes a pass. You continue to bet on the dice to win until a losing through occurs and then you switch your bets on to "Don't Pass." You simply switch when the dice are having a hot run and back again when the dice have a cold run.

### 7.5.5 The Place Betting System

A big time gamblers system which has even deceived some Craps operators. You place the maximum limit (although it can be less) on all of the six place bets: 4, 5, 6, 8, 9 and 10. If the limit is $\$ 200$ then this makes a total of $\$ 1,200$. As soon as one of these bets is won you collect on it and call off the remaining five place number bets. Yes, most casinos allow you to call off place number bets at any time. The theory is that the bettor has six numbers against the house's one, the 7 . There are 24 ways to make the place numbers and only 6 ways to make a 7 so the odds are worked out as 4 to 1 in favor of the gambler with the system.

### 7.5.6 The Right and Wrong Way System

In this system you place your bet (say $\$ 60$ ) on the "Don't Pass" line before the come out throw. The shooter is then supposed to throw a point on the come out roll, either $4,5,6,8,9$ or 10 . You then put $\$ 60$ on the points place number. The idea is if the shooter doesn't throw the point you have still won on the "Don't Pass" bet and you are still even. You've lost $\$ 60$ on the place number but won $\$ 60$ on Don't Pass. If the shooter makes his point you lose $\$ 60$ on the Don’t Pass but your up by between $\$ 48$ and $\$ 10$ depending on which place number it is.

## 8 More Probability and Expected Value Challenges

Question 4. Suppose you roll a die and receive the number of dollars that you roll. What is the fair value to play this game?

Question 5. If you flip a coin twice, what the probability that it will come up heads each time? What if you flip the coin 3 times? n times?

Question 6. Suppose we play a game in which you flip a coin until a heads comes up for the first time. What is the probability that the first heads will come up on the first flip? Second flip? Third flip? What about the $n^{\text {th }}$ flip?

Question 7. Play a game where you flip a coin until you get a heads. If the first heads comes up on the $k^{\text {th }}$ flip, then you win $2^{k}$ dollars. So if you get $H$ on the first try, you win \$2. If you get $T$ and then H, you win \$4. If you get TTTTH, you win \$32.

What is the least amount of money a player can win in this game? What is the most? How much would you be willing to pay to play this game? What is a fair amount to pay to play this game?

