1. Find
$$\int x^5 \ln(x)\ dx$$

2. Draw the direction field for the differential equation
$$\frac{dy}{dx} = y - x$$

3. If 8000 dollars is invested at 9 percent interest, find the value of the investment at the end of 9 years if the interest is compounded:
   (a) Annually
   (b) Monthly
   (c) Daily
   (d) Continuously

4. Solve the differential equation
$$\frac{dy}{dt} = 4y^6, \qquad y(0) = -6$$

5. A tank contains 90 kg of salt and 2000 L of water. A solution of a concentration 0.0225 kg of salt per liter enters a tank at the rate 9 L/min. The solution is mixed and drains from the tank at the same rate.
   (a) What is the concentration of our solution in the tank initially?
   (b) Find the amount of salt in the tank after 3 hours.
   (c) Find the concentration of salt in the solution in the tank as time approaches infinity.

6. A young person with no initial capital invests k dollars per year in a retirement account at an annual rate of return 0.1. Assume that investments are made continuously and that the return is compounded continuously.

   (a) Write a differential equation which models the rate of the change of the sum $S(t)$ with t in years.

   (b) Determine a formula for the sum $S(t)$.

   (c) What value of k will provide 3777000 dollars in 48 years?

7. Use Euler's method with step size 0.4 to estimate $y(2)$, where $y(x)$ is the solution of the initial-value problem

$$y' = -4x + y^2, \qquad y(0) = 0.$$

8. Suppose you have just poured a cup of freshly brewed coffee with temperature $90°C$ in a room where the temperature is $20°C$. Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Therefore, the temperature of the coffee, T(t), satisfies the differential equation

$$\frac{dT}{dt} = k(T - T_{\text{room}})$$

   where $T_{\text{room}} = 20$ is the room temperature, and k is some constant. Suppose it is known that the coffee cools at a rate of $1°C$ per minute when its temperature is $70°C$.

   (a) What is the limiting value of the temperature of the coffee?

   (b) What is the limiting value of the rate of cooling?

   (c) Find the constant k in the differential equation.

   (d) Use Euler's method with step size $h = 2$ minutes to estimate the temperature of the coffee after 10 minutes.

9. Compute the partial derivatives

$$z = \frac{7x}{\sqrt{x^2 + y^2}}$$

10. Find the critical point of the function $f(x, y) = x^2 + y^2 + 3xy - 20x$.

11. The function $f(x) = \sin(8x)$ has a Maclaurin series. Find the first 4 nonzero terms in the series, that is write down the Taylor polynomial with 4 nonzero terms.

12. Find the degree 3 Taylor polynomial $T_3(x)$ of function $f(x) = (7x + 214)^{5/4}$ at $a = 6$.

13. Determine the sum of the following series.

$$\sum_{n=1}^{\infty} \left(\frac{1^n + 7^n}{9^n}\right)$$

14. The following series are geometric series or a sum of two geometric series. Determine whether each series converges or not. For the series which converge, find the sum of the series.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{6^n}$

    (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2^n}$

    (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{7^{2n+1}}$

    (d) $\displaystyle\sum_{n=5}^{\infty} \frac{6^n}{7^n}$

    (e) $\displaystyle\sum_{n=1}^{\infty} \frac{9^n}{9^{n+4}}$

    (f) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n + 2^n}{7^n}$