X-Machines

1. Use dots and boxes to compute (using a $1 \leftarrow x$ machine).

   (a) $(x^3 - 3x^2 + 3x - 1) \div (x - 1)$

   (b) $(4x^3 - 14x^2 + 14x - 3) \div (2x - 3)$

   (c) $4x^5 - 2x^4 + 7x^3 + 6x - 1) \div (x^2 - x + 1)$

   (d) $(x^{10} - 1) \div (x^2 - 1)$

2. Use a $1|-1 \leftarrow 2$ machine.

   (a) What is the code for 5 in this machine?

   (b) What is the code for 20 in this machine?

   (c) What number has code $1|0|0|-1$?

   (d) Fraviana says that there is a value of x that is "appropriate" for this machine so that the boxes are labelled $\ldots, x^3, x^2, x, 1$. She is right, what is the "appropriate" value of x?

3. Use an $1 \leftarrow x$ machine to compute each of the following:

   (a) $\frac{x^2-1}{x-1}$

   (b) $\frac{x^4-1}{x-1}$

   (c) $\frac{x^6-1}{x-1}$

   (d) Will $x^{\text{even}} - 1$ always be a multiple of $x - 1$?

   (e) $\frac{x^6-1}{x+1}$

   (f) Will $x^{\text{even}} - 1$ always be a multiple of $x + 1$?

   (g) Explain why $2^{100} - 1$ must be a multiple of 3 and be a multiple of 5. Show that it is also a multiple of 33 and of 1023.

   (h) Is $x^7 + 1$ divisible by $x - 1$? Is it divisible by $x + 1$?

   (i) Is $x^{\text{odd}} + 1$ a multiple of $x - 1$? Of $x + 1$?

   (j) Explain why $2^{100} + 1$ is a multiple of 17. Show that $3^{100} + 1$ is a multiple of 41.

4. (a) Compute $(x^6 + x^5 + 5x^4 + 5x^3 + 9x^2 + 5x + 2) \div (x^2 + x + 2)$
   (b) Compute $(2x^6 - x^5 + 3x^4 - 10x^2 + 4x - 8) \div (x^2 - 2)$
   (c) Put x = 10 into your answer for a). What long division arithmetic problem have you solved? Do the same for your answer to b).

5. Remainders
   (a) Compute $(4x^4 - 7x^3 + 9x^2 - 3x - 1) \div (4x^2 - 3x + 3)$. Write out what this equals (correctly with the reminader).
   (b) Compute $x^4 \div (x^2 - 3)$
   (c) Compute $(5x^5 - 2x^4 + x^3 - x^2 + 7) \div (x^3 - 4x + 1)$

   HINT: Drawing dots and anti-dots in cells is tiresome. Instead of drawing 84 dots (as you will need to do at one point for problem c) it is easier just to write "84."