- You may use a calculator and you may have two 3×5 note cards of notes.
- Problem Solving
- Sets
- Properties of numbers and operations: closure, commutivity, associativity, distributivity, identity, inverses
- Exploding dots: bases, place value
- $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$
- Division algorithm
- Algorithms for operations: addition, subtraction, multiplication, long division
- Exponents
- Prime numbers
- Modular arithmetic
- Factors, greatest common divisor, least common multiple
- Fractions, rational numbers
- Decimals
- Division with $1 \leftarrow x$ machines (and other machines).
- Infinity
- Ratios, proportion and percent
- Irrational numbers
- Pythagorean Theorem

1. In solving 14×23 a student writes

$$14 \times 23 = (10 + 4) \times (20 + 3) = 200 + 80 + 30 + 12 = 322$$

This student is making use of

(a) The associative law

(b) The distributive law

(c) The commutativity law

(d) Unique factorization

2. Consider the statement: "The order in which one adds quantities does not matter." The mathematical axiom that expresses this notion is:

(a) $a + (b + c) = (a + b) + c$ for all numbers a, b, c.

(b) $a + b = b + a$ for all numbers a, b.

(c) $a(b + c) = ab + ac$ for all numbers a, b, c.

(d) Mathematical induction.

3. In computing 654 + 179 a student writes:

$$\begin{array}{rccc} & 6 & 5 & 4 \\ + & 1 & 7 & 9 \\ \hline & 7 & 12 & 13 = 833 \end{array}$$

Does this represent valid mathematical thinking? Explain.

4. What is the lowest common denominator of $\frac{1}{p^2q^4}$ and $\frac{1}{pq^6}$. Here p and q are two prime numbers.

5. Which of the following problems is about the division of fractions?

(a) Tom's driveway is 14 of a mile long. If Tom walks at a speed of three-and-a-half miles per hour, how long will it take him to walk the length of his driveway?

(b) One third of a field is planted with corn, one quarter with cabbages, and the rest with squash. What fraction of the field is planted with squash?

(c) John earns \$3500. He gives 30% of income to the IRS and the one third of what remains to his mother. How much money does John have remaining?

(d) One third of a town's men are married to two-fifths of the town's women. What is the ratio of married men to married women in this town?

6. If r is the radius of a circle and A is its area, which of the following cannot be true?

(a) A is rational

(b) A is irrational

(c) r is rational

(d) r is irrational

(e) r and A are both rational

(f) r and A are both irrational

7. The cost of a box of cereal is \$3.40. If the price suddenly goes up 120%, what will be the new cost of the cereal?

8. Which of the following systems is closed under the operation of taking square roots?

   (a) The counting numbers

   (b) The integers

   (c) The rationals

   (d) The reals

   (e) The complex numbers

9. The average number of passengers who use a certain airport each year is 350 thousand. A newspaper reported the number as 350 million. The number reported in the newspaper was how many times the actual number?

   (a) 10

   (b) 100

   (c) 1,000

   (d) 10,000

10. What is the expression $2^3 \times 2^2$ equivalent to?

    (a) 4

    (b) 2

    (c) $\frac{1}{2}$

    (d) $\frac{1}{4}$

11. Let $x = 5.194194194\ldots$. Represent x as $\frac{a}{b}$ where $a, b \in \mathbb{Z}$.

12. What is the simplified form of this expression

$$5^{2/3} \cdot 5^{7/4}$$

    (a) $5^{29/12}$

    (b) $5^{9/7}$

    (c) $5^{14/12}$

    (d) $5^{13/12}$