# Negatives

1. (a) What is the opposite of a hole?
   (b) What is the opposite of the opposite of the opposite of three piles?
   (c) What is the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of the opposite of ten hundred holes?

2. (a) Ali invented the notation "4 P + 5 H + 3H + 2 P = 2 H". What do you think he means by this? Do you like his notation?
   (b) Cuthbert writes "3 + 4 + 5 = 4". What do you think he means by his notation?

3. The statement: $-3+7=4$ reads "3 holes and 7 piles makes 4 piles," and the statement: $17+-6+-4+6+-20=-7$ reads "17 piles and 6 holes and 4 holes and 6 piles and 20 holes makes 7 holes." Translate each of the following, and give the answers!
   (a) $5+-9+2$
   (b) $3+-10+11-5$
   (c) $2+-2+2+-2+2+-2$
   (d) $-6+-1+-2+-3$

4. Pandi writes:

   $$------5+--2+----3+--------8$$

   This actually makes sense! What does it mean and what is the answer?

5. Interpret and evaluate the following:

   (a) $6 - (3 - 2)$

   (b) $4 + (1 - 3)$

   (c) $(3 - 9 - 1) - (8 + 2)$

   (d) $(1 + 2 - 4) - (3 + 2 - 4)$

   (e) $(2 - 3) - (5 - 2 - 3 + 7) - (-9 + 8 - 1) + (-2 - 3)$

   (f) $(16 - 16) - (17 - 17)$

6. What is $-x$ if:

   (a) $x$ is seven piles?

   (b) $x$ is seven holes?

   (c) $x$ is 50?

   (d) $x$ is $-50$ ?

7. Wanda is thinking of a number, which she calls W, but she refuses to tell us its value. But she does ask us to compute:

   $$(4 - W) - (2 - W)$$

   Without knowing her number do we have any chance of working this out?

8. Card Pile Trick

(a) Take 10 red cards and 10 black cards from a deck of cards. Shuffle your 20 cards and arbitrarily split them into two equal piles. Count the number of red cards in the left pile and the number of black cards in the right pile. What do you notice? Repeat this activity two more times.

(b) Shuffle your 20 cards and this time split them into a pile of 6 and a pile of 14 cards. Count the number of red cards in the small pile and count the number of black cards in the large pile. Take the (positive) difference of those two counts. Repeat this two more times.

(c) Shuffle the 20 cards again and this time split them into a pile of 9 cards and a pile of 11 cards. Count the number of red cards in the small pile, count the number of black cards in the large pile and take the (positive) difference of this count. What did you get? Repeat two more times. What do you notice?

(d) Complete the following table:

| **Small Pile** | **Large Pile** | **Different of R in Small and B in Large** |
|---|---|---|
| 10 | 10 | |
| 9 | 11 | |
| 8 | 12 | |
| 7 | 13 | |
| 6 | 14 | |
| 5 | 15 | |
| 4 | 16 | |
| 3 | 17 | |
| 2 | 18 | |
| 1 | 19 | |
| 0 | 20 | |

Any patterns?

(e) Suppose, in a game with 5 cards in the small pile and 15 cards in the large pile, I counted three red cards in the small pile. Complete the following table:

| | **Small Pile 5** | **Large Pile 15** |
|---|---|---|
| **# Reds** | 3 | |
| **# Blacks** | | |

What is the difference of counts of red cards in the small pile and black cards in the large pile?

(f) Suppose the small pile again has 5 cards and the large pile has 15 cards. Suppose it turns out there are $R$ red cards in the small pile. Complete the following table as an abstract exercise, writing formulas in each cell of the table:

| | **Small Pile 5** | **Large Pile 15** |
|---|---|---|
| **# Reds** | $R$ | |
| **# Blacks** | | |

What can you say about the difference between the number of red cards in the small pile and the number of black cards in the large pile? Does this match what you observed in part 8d of this question?

(g) Call the number of cards in the small pile $P$ and the number of cards in the large pile $20 - P$. (Why is this the correct formula for this?) Suppose there are $R$ red cards in the small pile. Complete the table again as the ultimate abstract exercise, writing formulas in each cell of the table:

| | **Small Pile** $P$ | **Large Pile** $20 - P$ |
|---|---|---|
| **# Reds** | $R$ | |
| **# Blacks** | | |

Can you write a formula for the difference of counts of the red cards in the small pile and black cards in the large pile? Does this formula match the data you obtained in part 8d of this question?