Problem Solving Methods

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One of the main points of problem solving is to learn techniques by just doing problems. So, lets start with a few problems and learn a few techniques.

Patience

1. Find a 9-digit number using each digit 1 through 9 once, such that the first n digits are divisible by n

2. Sheep and Wolves. On a 5×5 chessboard place 5 wolves (who move like chess queens) and 3 sheep so that the sheep are safe from being eaten by the wolves.

3. Triangle problem: arrange the numbers 1 through 6 into a "difference triangle" where each number in the row below is the difference of the two numbers above it. For example

   6 4 1
   2 3
   1

   almost works but it has two 1's and no 5.

   How about with 10 numbers? 15?

Try special cases (make up an easier problem!)

4. How many zeros are at the end of the number

$$100! = 100 \cdot 99 \cdot 98 \cdot 97 \cdots 3 \cdot 2 \cdot 1$$

5. The numbers 1 through 100 are written on the board. Take two numbers, u and v and erase them writing $uv + u + v$ in their place. This leaves 99 numbers. Keep doing this and after a while, there will only be one number left on the board. What are the possible numbers left?

Getting dirty

6. What is the smallest number that can not be written by subtracting a prime from a square. For example

$$1 = 4 - 3$$
$$2 = 9 - 7$$
$$3 = ?$$

   (How about the next smallest number?)

7. For every positive integer n, look at the number $n^3 - n$. The first few are here:

n	$n^3 - n$
1	0
2	6

   Keep filling out this chart. For at least the first few numbers in the $n^3 - n$ column, they should be divisible by 3.

   (a) Are all the numbers $n^3 - n$ divisible by 3?

   (b) If not, find one that is not. If so, show that this is always the case.

8. For every positive integer n, look at the number $n^5 - n$. The first few are here:

n	$n^5 - n$
1	0
2	40

   Keep filling out this chart. For at least the first few numbers in the $n^5 - n$ column, they should be divisible by 5.

   (a) Are all the numbers $n^5 - n$ divisible by 5?

   (b) If not, find one that is not. If so, show that this is always the case.

9. How many rectangles are in a 10×10 rectangle?