Irrationals

1. Show that the 17th root of 24 must be irrational.

2. Repeat our method of proof that "$\sqrt{2}$ is irrational" for the number $\sqrt{36}$. Determine where our proof fails.

3. Show that between any two rational numbers lies another rational number.

4. Show that between any two real numbers (they could be either rational or irrational) there corresponds a rational number.

5. Show that between any two real numbers there is an irrational number.

6. True or false. If true, prove it. If false, prove it (with a counter example) and explain when (if ever) it might be true.

   (a) The sum of two rationals is rational.

   (b) The sum of an irrational and a rationall is irrational.

   (c) The sum of two irrationals is irrational.

   (d) The product of two rationals is rational.

   (e) The product of two irrationals is irrational.

   (f) The product of a non-zero rational and an irrational is irrational.

   (g) The circumference of a circle is irrational.

   (h) The diagonal of a square is irrational.

   (i) If the circumference of a circle is rational, then the area is rational.

   (j) If the area of a circle is rational, then the circumference is rational.

   (k) If you divide one irrational number by another irrational, the result is always irrational.

   (l) If you divide a rational number by an irrational number, the result is always irrational.

   (m) If the radius of a circle is irrational, the area must be irrational.

   (n) The square root of an irrational number is irrational.

7. Prove that $\sqrt{2}+\sqrt{3}$ is irrational.
(HINT: Assume that $\sqrt{2}+\sqrt{3}=\frac{a}{b}$ and square both sides. What does this say about $\sqrt{6}$?)

8. Notice that $10^2 = 100$ and $10^3 = 1000$. There must be a number x so that $10^x = 700$. Prove that this number x cannot be rational.

9. Which natural numbers have rational square roots. (Find all of them and prove your answer is correct.)

10. Let $B = 0.01234567\ldots$.

    The "..." here is a bit confusing. The eleventh decimal place is meant to contain the "digit" 10, and the twelfth the "digit" 11, and so on. Of course, one would carry digits to form a decimal expansion with one number per place (like with our dots and boxes).

    (a) Show that $10B - \frac{1}{9} = B$.
    (HINT: What is the decimal expansion of $\frac{1}{9}$?)

    (b) Find the value of B and show that it is rational.
    This means, that when all the carrying is completed, the decimal expansion of B repeats!

    (c) What fraction is the decimal 0.00 01 02 03 04 05 06 07 ...?

    (d) What fraction is the decimal 0.000 001 002 003 004 005 ...?