Math 233 Warm Up Problems

September 29, 2009

1. Graph the following sets in $\mathbb{R}^2$.

   (a) $\{(x, y) | 1 < x < 3\}$
   (b) $\{(x, y) | 1 < y < 3\}$
   (c) $\{(x, y) | 1 < x, y < 3\}$
   (d) $\{(x, y) | 1 \leq x < 3\}$
   (e) $\{(x, y) | 1 < y \leq 3\}$
   (f) $\{(x, y) | 1 \leq x, y \leq 3\}$
   (g) $\{(x, y) | 1 \leq x \leq y \leq 5\}$

Lecture Problems

2. Determine if the following sets in $\mathbb{R}$ are closed, open, neither open nor closed, bounded or unbounded.

   (a) $(1, 10)$
   (b) $[2, 10]$
   (c) $[1, 10)$
   (d) $[1, 10] \cup [11, 21]$
   (e) $(1, \infty)$
   (f) $[1, \infty)$
   (g) $\{1, 2, 3, 4, \ldots\}$
   (h) $(1, 2) \cup (3, 4) \cup (5, 6) \cup \ldots$

Lecture Problems

2. Determine if the following sets in $\mathbb{R}$ are closed, open, neither open nor closed, bounded or unbounded.

   (a) $(1, 10)$ **Solution:** Open, bounded
   (b) $[2, 10]$ **Solution:** Closed, bounded
   (c) $[1, 10)$ **Solution:** Neither open nor closed, bounded
   (d) $[1, 10] \cup [11, 21]$ **Solution:** Closed, bounded
   (e) $(1, \infty)$ **Solution:** Open, unbounded
   (f) $[1, \infty)$ **Solution:** Closed, unbounded
   (g) $\{1, 2, 3, 4, \ldots\}$ **Solution:** Closed, unbounded
   (h) $(1, 2) \cup (3, 4) \cup (5, 6) \cup \ldots$ **Solution:** Open, unbounded

3. Determine if the following sets are closed, open, neither open nor closed, bounded or unbounded.
   (a) $\{(x, y) | 1 < x < 3\}$
   (b) $\{(x, y) | 1 < y < 3\}$
   (c) $\{(x, y) | 1 < x, y < 3\}$
   (d) $\{(x, y) | 1 \leq x < 3\}$
   (e) $\{(x, y) | 1 < y \leq 3\}$
   (f) $\{(x, y) | 1 \leq x, y \leq 3\}$
   (g) $\{(x, y) | 1 \leq x \leq y \leq 5\}$

3. Determine if the following sets are closed, open, neither open nor closed, bounded or unbounded.
   (a) $\{(x, y)|1 < x < 3\}$ **Solution:** Open, unbounded
   (b) $\{(x, y)|1 < y < 3\}$ **Solution:** Open, unbounded
   (c) $\{(x, y)|1 < x, y < 3\}$ **Solution:** Open, bounded
   (d) $\{(x, y)|1 \leq x < 3\}$ **Solution:** Neither open nor closed, unbounded
   (e) $\{(x, y)|1 < y \leq 3\}$ **Solution:** Neither open nor close, unbounded
   (f) $\{(x, y)|1 \leq x, y \leq 3\}$ **Solution:** Closed, bounded
   (g) $\{(x, y)|1 \leq x \leq y \leq 5\}$ **Solution:** Closed, bounded

4. Find the absolute maximum and absolute minimum of the function $f(x, y) = 2x^2 - 3xy + 6x$ on the rectangle $\{(x, y) | -1 \leq x \leq 2, -1 \leq y \leq 4\}$.

   (a) Find all critical points inside the rectangle.
   (b) Find the max/min of the function for $x = -1$, $-1 \leq y \leq 4$.
   (c) Find the max/min of the function for $x = 2$, $-1 \leq y \leq 4$.
   (d) Find the max/min of the function for $y = -1$, $-1 \leq x \leq 2$.
   (e) Find the max/min of the function for $y = 4$, $-1 \leq x \leq 2$.
   (f) What is the global max and min on the closed rectangle?

4. Find the absolute maximum and absolute minimum of the function $f(x, y) = 2x^2 - 3xy + 6x$ on the rectangle $\{(x, y) | -1 \leq x \leq 2, -1 \leq y \leq 4\}$.

   (a) Find all critical points inside the rectangle.
   Solution: $\nabla f = (4x - 3y + 6, 3x)$ and the only critical point is at $(0, 2)$.

   (b) Find the max/min of the function for $x = -1$, $-1 \leq y \leq 4$.
   Solution: Max of 8 at $(-1, 4)$. Min of -7 at $(-1, -1)$.

   (c) Find the max/min of the function for $x = 2$, $-1 \leq y \leq 4$.
   Solution: Max of 26 at $(2, -1)$. Min of -4 at $(2, 4)$.

   (d) Find the max/min of the function for $y = -1$, $-1 \leq x \leq 2$.
   Solution: Max of 26 at $(2, -1)$. Min of -7 at $(-1, -1)$. (You should have also found a point outside the range: $(-9/4, -1)$).

   (e) Find the max/min of the function for $y = 4$, $-1 \leq x \leq 2$.
   Solution: Max of 8 at $(-1, 4)$. Min of $-9/2$ at $(3/2, 4)$.

   (f) What is the global max and min on the closed rectangle?
   Solution: Max of 26. Min of -7.