Math 233 Warm Up Problems

September 28, 2009

Solutions

1. Find the maximum and minimum of the function $f(x) = x^3 - 3x^2 + 1$ on the interval $[1, 3]$.

   More importantly, what are the steps you followed to obtain your solution?

   Solution: Find critical points: $x = 0, 2$. We only care about $x = 2$ since $0 \notin [1, 3]$.

   Now plug all critical points and endpoints of interval into function: $f(1) = -1$, $f(2) = -3$, $f(3) = 1$.

   Thus we have a maximum of 1 at $x = 3$ and a minimum of $= 3$ at $x = 2$.

2. Find all local maximums and minimum of the function $f(x) = x^3 - 3x^2 + 1$.

   More importantly, what are the steps you followed to obtain your solution?

   Solution: Find all critical points: $x = 0, 2$. Test these critical points using the second derivative test $f''(0) = -6$ and is thus a local maximum. $f''(2) = 6$ and is thus a local minimum.

3. Find all points such that $\nabla f = \overrightarrow{0}$.

   (a) $f(x, y) = x^2 + 4y^2 - 4x$
   Solution: $(2, 0)$

   (b) $f(x, y) = x^3 + y^3 - 6xy$
   Solution: $(0, 0), (2, 2)$

   (c) $f(x, y) = xy + 2/x + 4/y$
   Solution: $(1, 2)$

   (d) $f(x, y) = (x - y)(xy - 1)$
   Solution: $(-1, -1), (1, 1)$

   (e) $f(x, y) = x^2y - 6y^2 - 4x^2$
   Solution: $(0, 0), (-6, 3), (6, 3)$

Lecture Problems

4. Find all critical points in Problem 3.

   Solution: This is the EXACTLY same question.