Math 233 Warm Up Problems

September 25, 2009

Solutions

1. From class we know that if $r = \sqrt{x^2+y^2+z^2}$ then $\nabla r = \dfrac{1}{r}(x,y,z)$.

   The chain rule tells us that if $g : \mathbb{R} \to \mathbb{R}$ then

   $$\nabla g = g'(r)\nabla r = g'(r)\frac{(x,y,z)}{r}$$

   Using the chain rule, find the gradient of the functions $g(r)$ where g is given.

   (a) $g(r) = e^r$, $\quad \nabla g = e^r \frac{X}{r} = \frac{e^r}{r}(x,y,z)$

   (b) $g(r) = e^{r^2}$, $\quad \nabla g = 2re^{r^2}\frac{X}{r} = 2e^r(x,y,z)$

   (c) $g(r) = \dfrac{1}{r^n}$, $\quad \nabla g = \frac{-n}{r^{n+1}}\frac{X}{r} = -\frac{n}{r^{n+2}}(x,y,x)$

   (d) $g(r) = 3r^2 - 2r + 1$, $\quad \nabla g = (6r-2)\frac{X}{r} = \frac{(6r-2)}{r}(x,y,x)$

2. For each vector field F below, find a function ψ such that $\nabla\psi = F$.

   (a) $F(x,y) = (1,1)$, $\quad \psi(x,y) = x + y$

   (b) $F(x,y) = (x,y)$, $\quad \psi(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$

   (c) $F(x,y) = (y,x)$, $\quad \psi(x,y) = yx$

   (d) $F(x,y) = (2xy, 4+x^2)$, $\quad \psi(x,y) = x^2y + 4y$

   (e) $F(x,y) = (x,x)$, $\quad \psi(x,y) =$ Not Possible!

   (f) $F(x,y) = (2x+y-2, x)$, $\quad \psi(x,y) = x^2 + xy - 2x$

   (g) $F(x,y) = ((x+1)e^{x+y}, xe^{x+y})$, $\quad \psi(x,y) = xe^{x+y}$

3. Find all points such that $\nabla f = \overrightarrow{0}$ for the given functions.

   (a) $f(x,y) = x^2 + y^2$
   Solution: $(0,0)$

   (b) $f(x,y) = x^2 - y^2$
   Solution: $(0,0)$

   (c) $f(x,y) = x + y$
   Solution: No solutions

   (d) $f(x,y) = xy - x^2 + y$
   Solution: $(-1,-2)$

   (e) $f(x,y) = xy - x^2 + y^3$
   Solution: $(0,0)$, $(-1/12, -1/6)$

(f) $f(x, y) = x^2y + yx^2 - y$
Solution: $(-1/\sqrt{2}, 0), (1/\sqrt{2}, 0)$

Lecture Problems

4. Find all critical points for the function in Problem 3.

   This is EXACTLY THE SAME QUESTION as Problem 3.