Math 233 Warm Up Problems
September 25, 2009

1. From class we know that if $r = \sqrt{x^2 + y^2 + z^2}$ then $\nabla r = \frac{1}{r}(x, y, z)$.
The chain rule tells us that if $g : \mathbb{R} \to \mathbb{R}$ then

$$\nabla g = g'(r)\nabla r = g'(r)\frac{(x, y, z)}{r}$$

Using the chain rule, find the gradient of the functions $g(r)$ where g is given.

(a) $g(r) = e^r$, $\quad \nabla g =$
(b) $g(r) = e^{r^2}$, $\quad \nabla g =$
(c) $g(r) = \frac{1}{r^n}$, $\quad \nabla g =$
(d) $g(r) = 3r^2 - 2r + 1$, $\quad \nabla g =$

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Using the chain rule, find the gradient of the functions $g(r)$ where g is given.

(a) $g(r) = e^r, \qquad \nabla g = e^r \frac{X}{r} = \frac{e^r}{r}(x, y, z)$

(b) $g(r) = e^{r^2}, \qquad \nabla g = 2re^{r^2} \frac{X}{r} = 2e^r(x, y, z)$

(c) $g(r) = \dfrac{1}{r^n}, \qquad \nabla g = \frac{-n}{r^{n+1}} \frac{X}{r} = -\frac{n}{r^{n+2}}(x, y, x)$

(d) $g(r) = 3r^2 - 2r + 1, \qquad \nabla g = (6r - 2)\frac{X}{r} = \frac{(6r-2)}{r}(x, y, x)$

2. For each vector field F below, find a function ψ such that $\nabla\psi = F$.

   (a) $F(x,y) = (1,1)$, $\psi(x,y) =$
   (b) $F(x,y) = (x,y)$, $\psi(x,y) =$
   (c) $F(x,y) = (y,x)$, $\psi(x,y) =$
   (d) $F(x,y) = (2xy, 4 + x^2)$, $\psi(x,y) =$
   (e) $F(x,y) = (x,x)$, $\psi(x,y) =$
   (f) $F(x,y) = (2x + y - 2, x)$, $\psi(x,y) =$
   (g) $F(x,y) = ((x+1)e^{x+y}, xe^{x+y})$, $\psi(x,y) =$

2. For each vector field F below, find a function ψ such that $\nabla\psi = F$.

(a) $F(x,y) = (1,1)$, $\psi(x,y) = x + y$
(b) $F(x,y) = (x,y)$, $\psi(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$
(c) $F(x,y) = (y,x)$, $\psi(x,y) = yx$
(d) $F(x,y) = (2xy, 4 + x^2)$, $\psi(x,y) = x^2y + 4y$
(e) $F(x,y) = (x,x)$, $\psi(x,y) =$ Not Possible!
(f) $F(x,y) = (2x + y - 2, x)$, $\psi(x,y) = x^2 + xy - 2x$
(g) $F(x,y) = ((x+1)e^{x+y}, xe^{x+y})$, $\psi(x,y) = xe^{x+y}$

3. Find all points such that $\nabla f = \overrightarrow{0}$ for the given functions.

(a) $f(x, y) = x^2 + y^2$
(b) $f(x, y) = x^2 - y^2$
(c) $f(x, y) = x + y$
(d) $f(x, y) = xy - x^2 + y$
(e) $f(x, y) = xy - x^2 + y^3$
(f) $f(x, y) = x^2y + yx^2 - y$

3. Find all points such that $\nabla f = \overrightarrow{0}$ for the given functions.

(a) $f(x, y) = x^2 + y^2$
Solution: $(0, 0)$

(b) $f(x, y) = x^2 - y^2$
Solution: $(0, 0)$

(c) $f(x, y) = x + y$
Solution: No solutions

(d) $f(x, y) = xy - x^2 + y$
Solution: $(-1, -2)$

(e) $f(x, y) = xy - x^2 + y^3$
Solution: $(0, 0)$, $(-1/12, -1/6)$

(f) $f(x, y) = x^2y + yx^2 - y$
Solution: $(-1/\sqrt{2}, 0), (1/\sqrt{2}, 0)$

Lecture Problems

4. Find all critical points for the function in Problem 3.

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4. Find all critical points for the function in Problem 3.
   This is EXACTLY THE SAME QUESTION as Problem 3.