Math 233 Warm Up Problems

September 25, 2009

1. From class we know that if $r = \sqrt{x^2 + y^2 + z^2}$ then $\nabla r = \frac{1}{r}(x, y, z)$.

   The chain rule tells us that if $g : \mathbb{R} \to \mathbb{R}$ then

   $$\nabla g = g'(r)\nabla r = g'(r)\frac{(x, y, z)}{r}$$

   Using the chain rule, find the gradient of the functions $g(r)$ where g is given.

   (a) $g(r) = e^r$, $\quad \nabla g =$

   (b) $g(r) = e^{r^2}$, $\quad \nabla g =$

   (c) $g(r) = \frac{1}{r^n}$, $\quad \nabla g =$

   (d) $g(r) = 3r^2 - 2r + 1$, $\quad \nabla g =$

2. For each vector field F below, find a function ψ such that $\nabla\psi = F$.

   (a) $F(x, y) = (1, 1)$, $\quad \psi(x, y) =$

   (b) $F(x, y) = (x, y)$, $\quad \psi(x, y) =$

   (c) $F(x, y) = (y, x)$, $\quad \psi(x, y) =$

   (d) $F(x, y) = (2xy, 4 + x^2)$, $\quad \psi(x, y) =$

   (e) $F(x, y) = (x, x)$, $\quad \psi(x, y) =$

   (f) $F(x, y) = (2x + y - 2, x)$, $\quad \psi(x, y) =$

   (g) $F(x, y) = ((x + 1)e^{x+y}, xe^{x+y})$, $\quad \psi(x, y) =$

3. Find all points such that $\nabla f = \overrightarrow{0}$ for the given functions.

   (a) $f(x, y) = x^2 + y^2$

   (b) $f(x, y) = x^2 - y^2$

   (c) $f(x, y) = x + y$

   (d) $f(x, y) = xy - x^2 + y$

   (e) $f(x, y) = xy - x^2 + y^3$

   (f) $f(x, y) = x^2y + yx^2 - y$

Lecture Problems

4. Find all critical points for the function in Problem 3.