Math 233 Warm Up Problems

September 24, 2009

1. *Linearizations*

(a) $f(x, y) = x^2 - xy + y$. Find the equation of the tangent plane to the graph at the point $(-2, 1, 7)$. Write your answer as a plane in the form $z =$.

$z =$

(b) $f(x, y) = x^2/(y + 1)$. Find the equation of the tangent plane to the graph at the point $(6, -3, -18)$. Write your answer as a plane in the form $z =$.

$z =$

1. *Linearizations*

(a) $f(x, y) = x^2 - xy + y$. Find the equation of the tangent plane to the graph at the point $(-2, 1, 7)$. Write your answer as a plane in the form $z =$.

$$z = -5x + 3y - 6$$

Note, you could also write this as

$$z = 7 - 5(x + 2) + 3(y - 1)$$

(b) $f(x, y) = x^2/(y + 1)$. Find the equation of the tangent plane to the graph at the point $(6, -3, -18)$. Write your answer as a plane in the form $z =$.

$$z = -9 - 6x - 9y$$

Note, you could also write this as

$$z = -18 - 6(x - 6) - 9(y + 3)$$

Lecture Problems

2. Using the formula for linearization (or first order Taylor polynomial):

$$L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

find the linearizations below

(a) $f(x, y) = x + y^2$ at the point $(1, 7)$.

$$L(x, y) =$$

(b) $f(x, y) = \sin(xy)$ at the point $(1/6, \pi)$.

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find the linearizations below

(a) $f(x, y) = x + y^2$ at the point $(1, 7)$.

$$L(x, y) = 50 + (x - 1) + 14(y - 7)$$

(b) $f(x, y) = \sin(xy)$ at the point $(1/6, \pi)$.

$$L(x, y) = \frac{1}{2} + \frac{\pi\sqrt{3}}{2}(x - 1/6) + \frac{\sqrt{3}}{12}(y - \pi)$$

3. Find the gradients of the functions

(a) $r = \sqrt{x^2 + y^2 + z^2}$, $\nabla r =$

(b) $f(x, y, z) = \sin r = \sin \sqrt{x^2 + y^2 + z^2}$, $\nabla f =$

3. Find the gradients of the functions

(a) $r = \sqrt{x^2 + y^2 + z^2}$, $\nabla r = \frac{1}{\sqrt{x^2+y^2+z^2}}(x, y, z) = \frac{(x,y,z)}{r}$

(b) $f(x, y, z) = \sin r = \sin\sqrt{x^2 + y^2 + z^2}$, $\nabla f = \frac{\cos r}{r}(x, y, z)$

4. Using the chain rule technique discussed in class, find the gradient of the functions. As usual, let $r = \sqrt{x^2 + y^2 + z^2}$.

   (a) $f(x, y, z) = e^r$, $\nabla f =$

   (b) $f(x, y, z) = e^{r^2}$, $\nabla f =$

   (c) $f(x, y, z) = \dfrac{1}{r^n}$, $\nabla f =$

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   (a) $f(x, y, z) = e^r$, $\nabla f = e^r \frac{X}{r} = \frac{e^r}{r}(x, y, z)$
   (b) $f(x, y, z) = e^{r^2}$, $\nabla f = 2re^{r^2} \frac{X}{r} = 2e^r(x, y, z)$
   (c) $f(x, y, z) = \dfrac{1}{r^n}$, $\nabla f = \frac{-n}{r^{n+1}} \frac{X}{r} = -\frac{n}{r^{n+2}}(x, y, x)$