

Math 233 Warm Up Problems

September 22, 2009

Solutions

1. Topic Outline:

- (a) Vectors: Cross product, dot product, norm
- (b) Lines and planes, distances between, equations of
- (c) Derivative of curves, lengths of curves
- (d) Graphs of functions, level curves and sets, slices
- (e) Partial derivatives, gradient, repeated partial derivatives, chain rule (for curves as in text)

Review Sessions: Tues: 3:30-5PM, McDonnell 162 Wed: 3:30-5PM, McDonnell 162

2. Find a unit vector orthogonal to both $(1, -1, 0)$ and $(0, 1, 2)$.

Solution: $(2/3, 2/3, -1/3)$

3. Find the equation of the line through $(1, 0, 6)$ and orthogonal to the plane $x + 3y + z = 5$.

Solution: $r(t) = (1, 0, 6) + t(1, 3, 1)$

4. Let $r(t) = (e^t, e^t \sin t, e^t \cos t)$. Find the equation of the plane that is normal to the curve at the point $(1, 0, 1)$.

Solution: $x + y + z = 2$

5. Let $r(t) = 1 + 2t, 1 + t, 1 - t + t^2 - t^3$. Find the point that the tangent line to the curve at $t = 0$ intersects the xy -plane.

Solution: $(3, 2, 0)$

6. Let $r(t) = (t^2, 2t, \ln t)$ for $1 \leq t \leq e$. Find the length of the curve.

Solution: e^2

7. Let $r(t) = (t^2, t, t^2 - 16t)$. When is speed maximum (and what is this max speed)?

Solution: $t = 4$, plug in to get max speed.

8. Let $f(x, y) = \sqrt{4x + 3y}$. Find all first and second order partial derivatives at the point $(4, 3)$.

Solution: $f_x(4, 3) = 2/5, f_y(4, 3) = 3/10, f_{xx}(4, 3) = -4/125, f_{yy}(4, 3) = -9/500, f_{xy}(4, 3) = -3/125, f_{yx}(4, 3) = -3/125,$

9. Let $z = f(x)g(y)$. If $f(0) = 1, g(0) = 2, f'(0) = 3$ and $g'(0) = 4$. What is $z_x(0, 0)$ and $z_y(0, 0)$?

Solution: $z_x(0, 0) = 6, z_y(0, 0) = 4$

10. Let $z = e^x \cos xy$. Find the equation of the plane tangent to the surface at $(1, \pi/2, 0)$. Find the z -intercept of this tangent plane.

Solution: $z = (-\pi e/2)(x - 1) - e(y - \pi/2)$, z -intercept at $(0, 0, \pi e)$.

11. Find an equation of the line parallel to $r(t) = (1, 0, 5) + t(2, 3, -7)$ and through the point $(0, 2, -1)$.

Solution: $L(t) = (0, 2, -1) + t(2, 3, -7)$

12. Let $A = (1, 0, 1)$ and $B = (1, -1, 0)$. Find the projection of B to A .

Solution: $(1/2, 0, 1/2)$

13. Let $A = (3, 1, 1)$ and $B = (1, -1, -2)$. Find the angle between A and B .

Solution: $\pi/2$

14. Find the distance from the origin to the plane $x - 2y - 2z - 1 = 0$.

Solution: $1/3$

15. Find the point on the curve $r(t) = t^3, -t^4, t^5$ where the tangent curve is nonzero and perpendicular to the plane $6x + 8y + 10z = 1$.

Solution: $t = -1$ and the point is $(-1, -1, -1)$

16. A particle start motion at the origin at $t = 0$, with initial velocity $v(0) = (1, -1, 0)$ and constant acceleration $a(t) = (0, 0, 1)$. Find the position function for the particle.

Solution: $r(t) = (t, -t, t^2/2)$

17. Suppose v and w are vectors with $v \times w = (-1, -5, 4)$ and $v = (3, 1, c)$. Find c .

Solution: 2. Use the fact that v is orthogonal to $v \times w$.

18. Find the intersection, if any, of the lines $r_1(t) = (1 + 2t, 3t, 2 - t)$ and $r_2(t) = (9 + 2t, 5 - 4t, 1 + 2t)$.

Solution: $(7, 9, -1)$

19. Find a vector normal to the plane through the points $(1, 0, 0)$, $(0, 4, 0)$, $(2, 3, 2)$.

Solution: $(8, 2, -7)$

20. Graph the following equations. Draw a bunch of level curves and slices too.

(a) $x + y + z = 1$

(b) $2x + y + z = 1$

(c) $z = x^2 + 4y^2$

(d) $z = x^2 + 4y^2 + 1$

(e) $z = x^2 + 4y^2 - 1$

(f) $z = x^2 - 4y^2 + 1$

(g) $z = -x^2 + 4y^2$

(h) $z^2 = x^2 + 4y^2$

(i) $z^2 = x^2 - 4y^2$

(j) $z^2 = x^2 - 4y^2 + 1$

(k) $z^2 = x^2 - 4y^2 - 1$

21. Let $r(t) = (t^2 + 2t, -2t, t^2 - t)$.

(a) Find speed at time $t = 0$.

Solution: 3

(b) Find the unit tangent vector when $t = 0$.

Solution: $(2/3, -2/3, -1/3)$

(c) What is the magnitude of acceleration when $t = 0$?

Solution: $2\sqrt{2}$

22. Find the length of $r(t) = (t^2/3 + 3t, 4t, t^2)$.

Solution: $16/3$

23. Find the point of intersection, if any, between the lines $r_1(t) = (1 + 3t, -1, 4 - 3t)$ and $r_2(t) = (1 + t, 1 - t, 2)$.

If they intersect, find the angle of intersection.

Solution: $(3, -1, 2)$, $\pi/3$.

24. Find the distance from the point $(1, -1, 2)$ to the plane $2x + y - z = 5$.

Solution: $\sqrt{6}$

25. Find all possible cosines of the angle between the planes $2x + y + z = 4$ and $3x - y - 3z - 1 = 0$.

Solution: $\pm 2/\sqrt{114}$

26. Parametrize a circle with center at $(12, 97)$ and radius 107.

Solution: One solution is $r(t) = (12 + 107 \cos t, 97 + 107 \sin t)$

27. Particle A and B move as follows:

$$r_A = (t, \sqrt{t}), \quad r_B = (t, t^2)$$

for $0 \leq t \leq 1$. For what values of t does A move faster than B ?

Solution: $0 < t < 16^{-1/3}$

28. Let $r(t) = (1, t, t^2)$. When $t = 2$, calculate the angle between the velocity vector and the acceleration vector.

Solution: $\cos^{-1}(4/\sqrt{17})$