Math 233 Warm Up Problems
September 22, 2009

1. Topic Outline:
    (a) Vectors: Cross product, dot product, norm
    (b) Lines and planes, distances between, equations of
    (c) Derivative of curves, lengths of curves
    (d) Graphs of functions, level curves and sets, slices
    (e) Partial derivatives, gradient, repeated partial derivatives, chain rule (for curves as in text)

Review Sessions:

Tues: 3:30-5PM, McDonnel 162

Wed: 3:30-5PM, McDonnel 162

2. Find a unit vector orthogonal to both $(1, -1, 0)$ and $(0, 1, 2)$.
3. Find the equation of the line through $(1, 0, 6)$ an orthogonal to the plane $x + 3y + z = 5$.
4. Let $r(t) = (e^t, e^t \sin t, e^t \cos t)$. Find the equation of the plane that is normal to the curve at the point $(1, 0, 1)$.
5. Let $r(t) = 1 + 2t, 1 + t, 1 - t + t^2 - t^3$. Find the point that the tangent line to the curve at $t = 0$ intersects the xy-plane.
6. Let $r(t) = (t^2, 2t, \ln t)$ for $1 \leq t \leq e$. Find the length of the curve.
7. Let $r(t) = t^2, t, t^2 - 16t)$. When is speed maximum (and what is this max speed)?

2. Find a unit vector orthogonal to both $(1,-1,0)$ and $(0,1,2)$.
   Solution: $(2/3, 2/3, -1/3)$
3. Find the equation of the line through $(1,0,6)$ an orthogonal to the plane $x+3y+z=5$.
   Solution: $r(t) = (1,0,6) + t(1,3,1)$
4. Let $r(t) = (e^t, e^t \sin t, e^t \cos t)$. Find the equation of the plane that is normal to the curve at the point $(1,0,1)$.
   Solution: $x+y+z=2$
5. Let $r(t) = 1+2t, 1+t, 1-t+t^2-t^3$. Find the point that the tangent line to the curve at $t=0$ intersects the xy-plane.
   Solution: $(3,2,0)$
6. Let $r(t) = (t^2, 2t, \ln t)$ for $1 \leq t \leq e$. Find the length of the curve.
   Solution: e^2
7. Let $r(t) = t^2, t, t^2 - 16t)$. When is speed maximum (and what is this max speed)?
   Solution: $t=4$, plug in to get max speed.

8. Let $f(x, y) = \sqrt{4x + 3y}$. Find all first and second order partial derivatives at the point $(4, 3)$.
9. Let $z = f(x)g(y)$. If $f(0) = 1$, $g(0) = 2$, $f'(0) = 3$ and $g'(0) = 4$. What is $z_x(0, 0)$ and $z_y(0, 0)$?
10. Let $z = e^x \cos xy$. Find the equation of the plane tangent to the surface at $(1, \pi/2, 0)$. Find the z-intercept of this tangent plane.
11. Find an equation of the line parallel to $r(t) = (1, 0, 5) + t(2, 3, -7)$ and through the point $(0, 2, -1)$.
12. Let $A = (1, 0, 1)$ and $B = (1, -1, 0)$. Find the projection of B to A.
13. Let $A = (3, 1, 1)$ and $B = (1, -1, -2)$. Find the angle between A and B.

8. Let $f(x,y) = \sqrt{4x+3y}$. Find all first and second order partial derivatives at the point $(4,3)$.
Solution: $f_x(4,3) = 2/5$, $f_y(4,3) = 3/10$, $f_{xx}(4,3) = -4/125$, $f_{yy}(4,3) = -9/500$, $f_{xy}(4,3) = -3/125$, $f_{yx}(4,3) = -3/125$,
9. Let $z = f(x)g(y)$. If $f(0) = 1$, $g(0) = 2$, $f'(0) = 3$ and $g'(0) = 4$. What is $z_x(0,0)$ and $z_y(0,0)$?
Solution: $z_x(0,0) = 6$, $z_y(0,0) = 4$
10. Let $z = e^x \cos xy$. Find the equation of the plane tangent to the surface at $(1, \pi/2, 0)$. Find the z-intercept of this tangent plane.
Solution: $z = (-\pi e/2)(x-1) - e(y - \pi/2)$, z-intercept at $(0,0,\pi e)$.
11. Find an equation of the line parallel to $r(t) = (1,0,5) + t(2,3,-7)$ and through the point $(0,2,-1)$.
Solution: $L(t) = (0,2,-1) + t(2,3,-7)$
12. Let $A = (1,0,1)$ and $B = (1,-1,0)$. Find the projection of B to A.
Solution: $(1/2, 0, 1/2)$
13. Let $A = (3,1,1)$ and $B = (1,-1,-2)$. Find the angle between A and B.
Solution: $\pi/2$

14. Find the distance from the origin to the plane $x - 2y - 2z - 1 = 0$.
15. Find the point on the curve $r(t) = t^3, -t^4, t^5$ where the tangent curve is nonzero and perpendicular to the plane $6x + 8y + 10z = 1$.
16. A particle start motion at the origin at $t = 0$, with initial velocity $v(0) = (1, -1, 0)$ and constant acceleration $a(t) = (0, 0, 1)$. Find the position function for the particle.
17. Suppose v and w are vectors with $v \times w = (-1, -5, 4)$ and $v = (3, 1, c)$. Find c.
18. Find the intersection, if any, of the lines $r_1(t) = (1 + 2t, 3t, 2 - t)$ and $r_2(t) = (9 + 2t, 5 - 4t, 1 + 2t)$.
19. Find a vector normal to the plane though the points $(1, 0, 0)$, $(0, 4, 0)$, $(2, 3, 2)$.

14. Find the distance from the origin to the plane $x - 2y - 2z - 1 = 0$.
Solution: $1/3$

15. Find the point on the curve $r(t) = t^3, -t^4, t^5$ where the tangent curve is nonzero and perpendicular to the plane $6x + 8y + 10z = 1$.
Solution: $t = -1$ and the point is $(-1, -1, -1)$

16. A particle start motion at the origin at $t = 0$, with initial velocity $v(0) = (1, -1, 0)$ and constant acceleration $a(t) = (0, 0, 1)$. Find the position function for the particle.
Solution: $r(t) = (t, -t, t^2/2)$

17. Suppose v and w are vectors with $v \times w = (-1, -5, 4)$ and $v = (3, 1, c)$. Find c.
Solution: 2. Use the face that v is orthogonal to $v \times w$.

18. Find the intersection, if any, of the lines $r_1(t) = (1 + 2t, 3t, 2 - t)$ and $r_2(t) = (9 + 2t, 5 - 4t, 1 + 2t)$.
Solution: $(7, 9, -1)$

19. Find a vector normal to the plane though the points $(1, 0, 0)$, $(0, 4, 0)$, $(2, 3, 2)$.
Solution: $(8, 2, -7)$

20. Graph the following equations. Draw a bunch of level curves and slices too.

(a) $x + y + z = 1$
(b) $2x + y + z = 1$
(c) $z = x^2 + 4y^2$
(d) $z = x^2 + 4y^2 + 1$
(e) $z = x^2 + 4y^2 - 1$
(f) $z = x^2 - 4y^2 + 1$
(g) $z = -x^2 + 4y^2$
(h) $z^2 = x^2 + 4y^2$
(i) $z^2 = x^2 - 4y^2$
(j) $z^2 = x^2 - 4y^2 + 1$
(k) $z^2 = x^2 - 4y^2 - 1$

21. Let $r(t) = (t^2 + 2t, -2t, t^2 - t)$.
    (a) Find speed at time $t = 0$.
    (b) Find the unit tangent vector when $t = 0$.
    (c) What is the magnitude of acceleration when $t = 0$?
22. Find the length of $r(t) = (t^2/3 + 3t, 4t, t^2)$.
23. Find the point of intersection, if any, between the lines $r_1(t) = (1 + 3t, -1, 4 - 3t)$ and $r_2(t) = (1 + t, 1 - t, 2)$. If they intersect, find the angle of intersection.

21. Let $r(t) = (t^2 + 2t, -2t, t^2 - t)$.
   (a) Find speed at time $t = 0$.
   Solution: 3
   (b) Find the unit tangent vector when $t = 0$.
   Solution: $(2/3, -2/3, -1/3)$
   (c) What is the magnitude of acceleration when $t = 0$?
   Solution: $2\sqrt{2}$

22. Find the length of $r(t) = (t^2/3 + 3t, 4t, t^2)$.
   Solution: $16/3$

23. Find the point of intersection, if any, between the lines $r_1(t) = (1 + 3t, -1, 4 - 3t)$ and $r_2(t) = (1 + t, 1 - t, 2)$. If they intersect, find the angle of intersection.
   Solution: $(3, -1, 2)$, $\pi/3$.

24. Find the distance from the point $(1, -1, 2)$ to the plane $2x + y - z = 5$.
25. Find all possible cosines of the angle between the planes $2x + y + z = 4$ and $3x - y - 3z - 1 = 0$.
26. Parametrize a circle with center at $(12, 97)$ and radius 107.
27. Particle A and B move as follows:

$$r_A = (t, \sqrt{t}), \qquad r_B = (t, t^2)$$

for $0 \leq t \leq 1$. For what values of t does A move faster than B?
28. Let $r(t) = (1, t, t^2)$. When $t = 2$, calculate the angle between the velocity vector and the acceleration vector.

24. Find the distance from the point $(1, -1, 2)$ to the plane $2x + y - z = 5$.
Solution: $\sqrt{6}$

25. Find all possible cosines of the angle between the planes $2x + y + z = 4$ and $3x - y - 3z - 1 = 0$.
Solution: $\pm 2/\sqrt{114}$

26. Parametrize a circle with center at $(12, 97)$ and radius 107.
Solution: One solution is $r(t) = (12 + 107\cos t, 97 + 107\sin t)$

27. Particle A and B move as follows:

$$r_A = (t, \sqrt{t}), \qquad r_B = (t, t^2)$$

for $0 \leq t \leq 1$. For what values of t does A move faster than B?
Solution: $0 < t < 16^{-1/3}$

28. Let $r(t) = (1, t, t^2)$. When $t = 2$, calculate the angle between the velocity vector and the acceleration vector.
Solution: $\cos^{-1}(4/\sqrt{17})$