

Math 233 Warm Up Problems

September 21, 2009

Solutions

1. Find the tangent planes

(a) $x^3yz = -4$ at the point $(1, 2, -2)$

Solution:

$$-12x - 2y + 2z = -20$$

(b) $x^3 + xy^2 + yz^2 = 14$ at the point $(2, 1, -2)$

Solution:

$$12x + 8y - 4z = 42$$

(c) Tangent to the graph of $f(x, y) = x^2 - 2xy^2$ at the point $(-1, 3, 19)$.

Solution:

$$-20x + 12y - z = 37$$

(d) Tangent to the graph of $f(x, y) = (x + 1)/(y - 1)$ at the point $(1, -3, -1/2)$

Solution:

$$-\frac{1}{4}x - \frac{1}{8}y - z = \frac{5}{8}$$

2. Suppose $f(5, 9) = 12$, $f_x(5, 9) = 3$ and $f_y(5, 9) = -2$. Make your best estimates for f if:

(a) You move one unit in the E_1 direction. **Solution:** $f(6, 9) \approx 15$.

(b) You move one unit in the $(-1, 0)$ direction. **Solution:** $f(4, 9) \approx 9$.

(c) You move one unit in the $(0, 1)$ direction. **Solution:** $f(5, 10) \approx 10$.

(d) You move one unit in the $(0, -1)$ direction. **Solution:** $f(5, 8) \approx 14$.

(e) You move one unit in the $(1, 1)$ direction. **Solution:** NOT: $f(6, 10) \approx 12 + 3 - 2 = 13$

Solution: $f(5 + 1/\sqrt{2}, 9 + 1/\sqrt{2}) \approx 12 + \frac{1}{\sqrt{12}}$

(f) You move one unit in the $(1, -3)$ direction. **Solution:** NOT: $f(6, 6) \approx 12 + 3 + 6 = 21$

Solution: $f(5 + 1/\sqrt{10}, 9 - 3/\sqrt{10}) \approx 12 + \frac{9}{\sqrt{10}}$

(g) You move one unit in the $(-2, 3)$ direction. **Solution:** NOT: $f(7, 12) \approx 12 - 6 - 6 = 0$

Solution: $f(5 - 2/\sqrt{13}, 9 + 3/\sqrt{13}) \approx 12 - \frac{12}{\sqrt{13}}$

Lecture Problems

3. Find the directional derivatives

(a) Let $f(x, y) = x + 2y$, $P = (1, 2)$ and $A = (3, 2)$.

$$D_A f(P) = \frac{7}{\sqrt{13}}$$

(b) Let $f(x, y) = 2x - y + z^2$, $P = (2, 1, 3)$ and $A = (1, 1, 2)$.

$$D_A f(P) = -\frac{11}{\sqrt{6}}$$

(c) Let $f(x, y) = e^{x+yz}$, $P = (0, 0, 1)$ and $A = (1, 2, 1)$.

$$D_A f(P) = \frac{5}{\sqrt{6}}$$

(d) Let $f(x, y) = e^{x+yz}$, $P = (0, 0, 1)$ and $A = (-1, -2, -1)$.

$$D_A f(P) = -\frac{5}{\sqrt{6}}$$

4. Captain Picard has the Enterprise located on the sunny side of Mercury at position $(1, 1, 1)$ where the temperature at location (x, y, z) is

$$T(x, y, z) = e^{-x^2-2y^2-3z^2}$$

What direction should Captain Picard travel to decrease the temperature the most rapidly?

Solution:

$$\nabla f = 2e^{-6}(-1, -2, -3)$$

Captain Picard should travel in the direction $(1, 2, 3)$.