Math 233 Warm Up Problems

September 21, 2009

1. Find the tangent planes
   (a) $x^3yz = -4$ at the point $(1, 2, -2)$
   (b) $x^3 + xy^2 + yz^2 = 14$ at the point $(2, 1, -2)$
   (c) Tangent to the graph of $f(x, y) = x^2 - 2xy^2$ at the point $(-1, 3, 19)$.
   (d) Tangent to the graph of $f(x, y) = (x + 1)/(y - 1)$ at the point $(1, -3, -1/2)$

1. Find the tangent planes

   (a) $x^3yz = -4$ at the point $(1, 2, -2)$
   Solution:

   $$-12x - 2y + 2z = -20$$

   (b) $x^3 + xy^2 + yz^2 = 14$ at the point $(2, 1, -2)$
   Solution:

   $$12x + 8y - 4z = 42$$

   (c) Tangent to the graph of $f(x, y) = x^2 - 2xy^2$ at the point $(-1, 3, 19)$.
   Solution:

   $$-20x + 12y - z = 37$$

   (d) Tangent to the graph of $f(x, y) = (x + 1)/(y - 1)$ at the point $(1, -3, -1/2)$
   Solution:

   $$-\frac{1}{4}x - \frac{1}{8}y - z = \frac{5}{8}$$

2. Suppose $f(5,9) = 12$, $f_x(5,9) = 3$ and $f_y(5,9) = -2$. Make your best estimates for f if:
   (a) You move one unit in the E_1 direction.
   (b) You move one unit in the $(-1,0)$ direction.
   (c) You move one unit in the $(0,1)$ direction.
   (d) You move one unit in the $(0,-1)$ direction.
   (e) You move one unit in the $(1,1)$ direction.
   (f) You move one unit in the $(1,-3)$ direction.
   (g) You move one unit in the $(-2,3)$ direction.

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   (a) You move one unit in the E_1 direction.
   Solution: $f(6,9) \approx 15$.
   (b) You move one unit in the $(-1,0)$ direction.
   Solution: $f(4,9) \approx 9$.
   (c) You move one unit in the $(0,1)$ direction.
   Solution: $f(5,10) \approx 10$.
   (d) You move one unit in the $(0,-1)$ direction.
   Solution: $f(5,8) \approx 14$.
   (e) You move one unit in the $(1,1)$ direction.
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   (e) You move one unit in the $(1,1)$ direction.
   Solution: NOT: $f(6,10) \approx 12 + 3 - 2 = 13$

   (f) You move one unit in the $(1,-3)$ direction.
   Solution: NOT: $f(6,6) \approx 12 + 3 + 6 = 21$

   (g) You move one unit in the $(-2,3)$ direction.
   Solution: NOT: $f(7,12) \approx 12 - 6 - 6 = 0$

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   (e) You move one unit in the $(1,1)$ direction.
   Solution: NOT: $f(6,10) \approx 12 + 3 - 2 = 13$
   Solution: $f(5 + 1/\sqrt{2}, 9 + 1/\sqrt{2}) \approx 12 + \frac{1}{\sqrt{12}}$

   (f) You move one unit in the $(1,-3)$ direction.
   Solution: NOT: $f(6,6) \approx 12 + 3 + 6 = 21$
   Solution: $f(5 + 1/\sqrt{10}, 9 - 3/\sqrt{10}) \approx 12 + \frac{9}{\sqrt{10}}$

   (g) You move one unit in the $(-2,3)$ direction.
   Solution: NOT: $f(7,12) \approx 12 - 6 - 6 = 0$
   Solution: $f(5 - 2/\sqrt{13}, 9 + 3/\sqrt{13}) \approx 12 - \frac{12}{\sqrt{13}}$

Lecture Problems

3. Find the directional derivatives

   (a) Let $f(x, y) = x + 2y$, $P = (1, 2)$ and $A = (3, 2)$.

   $$D_A f(P) =$$

   (b) Let $f(x, y) = 2x - y + z^2$, $P = (2, 1, 3)$ and $A = (1, 1, 2)$.

   $$D_A f(P) =$$

   (c) Let $f(x, y) = e^{x+yz}$, $P = (0, 0, 1)$ and $A = (1, 2, 1)$.

   $$D_A f(P) =$$

   (d) Let $f(x, y) = e^{x+yz}$, $P = (0, 0, 1)$ and $A = (-1, -2, -1)$.

   $$D_A f(P) =$$

Lecture Problems

3. Find the directional derivatives

   (a) Let $f(x, y) = x + 2y$, $P = (1, 2)$ and $A = (3, 2)$.

   $$D_A f(P) = \frac{7}{\sqrt{13}}$$

   (b) Let $f(x, y) = 2x - y + z^2$, $P = (2, 1, 3)$ and $A = (1, 1, 2)$.

   $$D_A f(P) = -\frac{11}{\sqrt{6}}$$

   (c) Let $f(x, y) = e^{x+yz}$, $P = (0, 0, 1)$ and $A = (1, 2, 1)$.

   $$D_A f(P) = \frac{5}{\sqrt{6}}$$

   (d) Let $f(x, y) = e^{x+yz}$, $P = (0, 0, 1)$ and $A = (-1, -2, -1)$.

   $$D_A f(P) = -\frac{5}{\sqrt{6}}$$

4. Captain Picard has the Enterprise located on the sunny side of Mercury at position $(1, 1, 1)$ where the temperature at location (x, y, z) is

$$T(x, y, z) = e^{-x^2-2y^2-3z^2}$$

What direction should Captain Picard travel to decrease the temperature the most rapidly?

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$$T(x, y, z) = e^{-x^2-2y^2-3z^2}$$

What direction should Captain Picard travel to decrease the temperature the most rapidly?
Solution:

$$\nabla f = 2e^{-6}(-1, -2, -3)$$

Captain picard should travel in the direction $(1, 2, 3)$.