

Math 233 Warm Up Problems

September 15, 2009

1. Find the gradients of the functions

(a)

$$f(x, y) = \frac{x}{x + y^2} \quad \nabla f =$$

(b)

$$f(x, y, z) = \frac{x}{y + z^2} \quad \nabla f =$$

2. Suppose you have a function $f(x, y)$ and you know that

$$f(4, 7) = 10 \quad \frac{\partial f}{\partial x}(4, 7) = 3 \quad \frac{\partial f}{\partial y}(4, 7) = -2$$

Determine your best estimates for the following

$$f(5, 7) \approx \quad f(3, 7) \approx$$

$$f(4, 8) \approx \quad f(4, 6) \approx$$

1. Find the gradients of the functions
(a)

$$f(x, y) = \frac{x}{x + y^2} \quad \nabla f = \left(\frac{y^2}{(y^2 + x)^2}, -\frac{2xy}{(y^2 + x)^2} \right)$$

(b)

$$f(x, y, z) = \frac{x}{y + z^2} \quad \nabla f = \left(\frac{1}{z^2 + y}, -\frac{x}{(z^2 + y)^2}, -\frac{2xz}{(z^2 + y)^2} \right)$$

2. Suppose you have a function $f(x, y)$ and you know that

$$f(4, 7) = 10 \quad \frac{\partial f}{\partial x}(4, 7) = 3 \quad \frac{\partial f}{\partial y}(4, 7) = -2$$

Determine your best estimates for the following

$$\begin{aligned} f(5, 7) &\approx 13 & f(3, 7) &\approx 7 \\ f(4, 8) &\approx 8 & f(4, 6) &\approx 12 \end{aligned}$$

Lecture Problems

3. $f(x, y) = x^2 y^3$

$$f_{xx} = \quad \quad \quad f_{xy} =$$

$$f_{yx} = \quad \quad \quad f_{yy} =$$

4. $f(x, y) = x \sin(y)$

$$D_1 D_1 f = \quad \quad \quad D_2 D_1 f =$$

$$D_1 D_2 f = \quad \quad \quad D_2 D_2 f =$$

Lecture Problems

3. $f(x, y) = x^2 y^3$

$$f_{xx} = 2y^3$$

$$f_{xy} = 6xy^2$$

$$f_{yx} = 6xy^2$$

$$f_{yy} = 6xy$$

4. $f(x, y) = x \sin(y)$

$$D_1 D_1 f = 0$$

$$D_2 D_1 f = \cos y$$

$$D_1 D_2 f = \cos y$$

$$D_2 D_2 f = -x \sin y$$

5. $f(x, y) = \cos(xy)$

$$\frac{\partial^2 f}{\partial x^2} f =$$

$$\frac{\partial^2 f}{\partial y \partial x} f =$$

$$\frac{\partial^2 f}{\partial x \partial y} f =$$

$$\frac{\partial^2 f}{\partial y^2} f =$$

6. Let

$$f(x, y, z) = x + y^2 + z^3$$

$$r(t) = (\sin t, \cos t, t)$$

$$(f \circ r)'(t) =$$

5. $f(x, y) = \cos(xy)$

$$\frac{\partial^2 f}{\partial x^2} f = -y^2 \cos xy$$

$$\frac{\partial^2 f}{\partial y \partial x} f = -\sin xy - xy \cos xy$$

$$\frac{\partial^2 f}{\partial x \partial y} f = -\sin xy - xy \cos xy$$

$$\frac{\partial^2 f}{\partial y^2} f = -x^2 \cos xy$$

6. Let

$$f(x, y, z) = x + y^2 + z^3$$

$$r(t) = (\sin t, \cos t, t)$$

$$(f \circ r)'(t) = -2 \cos t \sin t + \cos t + 3t^2$$