

Math 233 Warm Up Problems

September 14, 2009

Solutions

1. Draw some graphs and level curves

(a) $x^2 + y^2 + 16x^2 = 36$

(b) $z = x + y^2$

(c) $z = x - y^2$

Lecture Problems

2. Compute the partial derivatives

(a) Find $\partial z / \partial x$ and $\partial z / \partial y$ if $z = \sin(xy)$.

Solution:

$$\frac{\partial z}{\partial x} = y \cos xy \qquad \frac{\partial z}{\partial y} = x \cos xy$$

(b) Find f_x and f_z if $f(x, y, z) = xyz^2 + e^{yz}$.

Solution:

$$f_x = yz^2 \qquad f_z = 2xyz + ye^{yz}$$

(c) Find $D_1 f$ and D_2 if $f(x, y, z) = \frac{xy}{z^2}$

Solution:

$$D_1 f = \frac{y}{z^2} \qquad D_2 f = -\frac{2xy}{z^3}$$

3. (a) Let $f(x, y) = \frac{x^4+1}{y^5}$. Find ∇f .

Solution:

$$\nabla f = \left(\frac{4x^3}{y^5}, -\frac{5(x^4+1)}{y^6} \right)$$

(b) Let $f(x, y, z) = xyz$. Find ∇f .

Solution:

$$\nabla f = (yz, xz, xy)$$

(c) Let $f(x, y, z, w) = xyzw \sin x$. Find ∇f .

Solution:

$$\nabla f = (yzw \sin x + xyzw \cos(x), xzw \sin x, xyw \sin x, xyz \sin x)$$

4. Let $f(x, y) = x^2 - y^2$. Let $P = (3, -1)$, $Q = (2.8, -1)$ and $R = (3, -0.7)$ Compute $\nabla f(P)$

Solution:

$$\nabla f = (6, 2)$$

- (a) Use the gradient to describe by how much the function value changes when you move from point P to point Q .

Solution: We expect $f(Q)$ to be approximately $6(-0.2) = -1.2$ more than $f(P)$.

- (b) Use the gradient to describe by how much the function value changes when you move from point P to point R .

Solution: We expect $f(R)$ to be approximately $2(0.3) = 0.6$ more than $f(P)$.