Math 233 Warm Up Problems
September 14, 2009
1. Draw some graphs and level curves
   (a) $x^2 + y^2 + 16x^2 = 36$
   (b) $z = x + y^2$
   (c) $z = x - y^2$
Lecture Problems

2. Compute the partial derivatives
   (a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = \sin(xy)$.
   Solution:
   $$\frac{\partial z}{\partial x} = y \cos xy \quad \frac{\partial z}{\partial y} = x \cos xy$$

   (b) Find $f_x$ and $f_z$ if $f(x, y, z) = xyz^2 + e^{yz}$.
   Solution:
   $$f_x = yz^2 \quad f_z = 2xyz + ye^{yz}$$

   (c) Find $D_1 f$ and $D_2$ if $f(x, y, z) = \frac{xy}{z^2}$
   Solution:
   $$D_1 f = \frac{y}{z^2} \quad D_2 f = -\frac{2xy}{z^3}$$
3. (a) Let \( f(x, y) = \frac{x^4 + 1}{y^5} \). Find \( \nabla f \).
Solution:
\[
\nabla f = \left( \frac{4x^3}{y^5}, -\frac{5(x^4 + 1)}{y^6} \right)
\]

(b) Let \( f(x, y, z) = xyz \). Find \( \nabla f \).
Solution:
\[
\nabla f = (yz, xz, xy)
\]

(c) Let \( f(x, y, z, w) = xyzw \sin x \). Find \( \nabla f \).
Solution:
\[
\nabla f = (yzw \sin x + xyzw \cos(x), xzw \sin x, xyw \sin x, xyz \sin x)
\]
4. Let \( f(x, y) = x^2 - y^2 \). Let \( P = (3, -1) \), \( Q = (2.8, -1) \) and \( R = (3, -0.7) \) Compute \( \nabla f(P) \)

**Solution:**

\[ \nabla f = (6, 2) \]

(a) Use the gradient to describe by how much the function value changes when you move from point \( P \) to point \( Q \).

**Solution:** We expect \( f(Q) \) to be approximately \( 6(-0.2) = -1.2 \) more than \( f(P) \).

(b) Use the gradient to describe by how much the function value changes when you move from point \( P \) to point \( R \).

**Solution:** We expect \( f(R) \) to be approximately \( 2(0.3) = 0.6 \) more than than \( f(P) \).