Math 233 Warm Up Problems

September 10, 2009

1. Let $r(t) = (t^2, t^3)$.
   (a) When $t = 1$, find the angle between the the velocity vector and the acceleration vector.
   (b) Find the length of the curve from $t = 0$ to $t = t$.
   (c) Find $\frac{ds}{dt}$ for your arc length function.

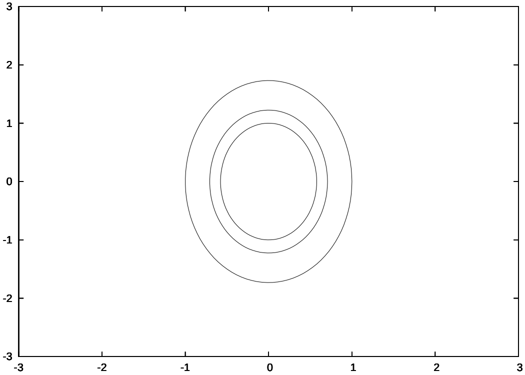
1. Let $r(t) = (t^2, t^3)$.
   (a) When $t = 1$, find the angle between the the velocity vector and the acceleration vector.
   Solution:
   $$\begin{aligned} r'(1) =& (2,3), \qquad r''(1) = (2,6) \\ \cos\theta =& \frac{(2,3)\cdot(2,6)}{\|(2,3)\|\|(2,6)\|} = \frac{11}{\sqrt{130}} \end{aligned}$$
   (b) Find the length of the curve from $t = 0$ to $t = t$.
   Solution:
   $$\begin{aligned} s(t) &= \int_0^t \|r'(u)\|\, du = \int_0^t \sqrt{9u^4 + 4u^2}\, du \\ &= \int_0^t u\sqrt{9u^2+4}\, du \qquad \text{(use substitution, } w = 9u^2 + 4) \\ &= \frac{1}{27}(9u^2+4)^{3/2}\Big|_0^t = \frac{1}{27}(9t^2+4)^{3/2} - \frac{8}{27} \end{aligned}$$
   (c) Find $\frac{ds}{dt}$ for your arc length function.
   Solution:
   $$s'(t) = \|r'(t)\| = \sqrt{9t^4 + 4t^2}$$

Lecture Problems

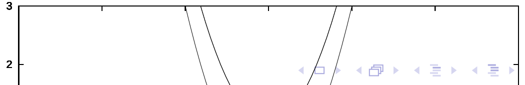
2. Let $z = f(x, y) = x^2 + 3y^2$.
   (a) Draw a single graph (in the xy-plane) with the following slices: $z = -3, z = -2, z = -1, z = 0, z = 1, z = 2, z = 3$.
   (b) Draw a single graph (in the yz-plane) with the following slices: $x = -3, x = -2, x = -1, x = 0, x = 1, x = 2, x = 3$.
   (c) Draw a single graph (in the xz-plane) with the following slices: $y = -3, y = -2, y = -1, y = 0, y = 1, y = 2, y = 3$.

Lecture Problems

2. Let $z = f(x, y) = x^2 + 3y^2$.
   (a) Draw a single graph (in the xy-plane) with the following slices: $z = -3, z = -2, z = -1, z = 0, z = 1, z = 2, z = 3$.
   Solution:



   (b) Draw a single graph (in the yz-plane) with the following slices: $x = -3, x = -2, x = -1, x = 0, x = 1, x = 2, x = 3$.
   Solution:



3. Draw the following graphs in $\mathbb{R}^3$. As as many (or as few) slices in each direction that you need.

   (a) $f(x,y) = 4 - 3x + 2y$
   (b) $f(x,y) = \sqrt{100 - x^2 - y^2}$
   (c) $f(x,y) = 3x - 7y$
   (d) $f(x,y) = x^2 + xy$
   (e) $f(x,y) = x/y$
   (f) $4x^2 + y^2 = 16$
   (g) $\frac{x}{4} = \frac{y^2}{4} + \frac{z^2}{9}$
   (h) $z = x^2$
   (i) $\frac{x^2}{9} + \frac{y^2}{12} + \frac{z^2}{9} = 1$
   (j) $4x^2 - 3y^2 + 2z^2 = 0$
   (k) $x^2 + y^2 - 2x = 0$