1. Let $r(t) = (t^2, t^3)$.

(a) When $t = 1$, find the angle between the the velocity vector and the acceleration vector.

(b) Find the length of the curve from $t = 0$ to $t = t$.

(c) Find $\frac{ds}{dt}$ for your arc length function.
1. Let $r(t) = (t^2, t^3)$.

(a) When $t = 1$, find the angle between the velocity vector and the acceleration vector.

**Solution:**

$$r'(1) = (2, 3), \quad r''(1) = (2, 6)$$

$$\cos \theta = \frac{\langle 2, 3 \rangle \cdot \langle 2, 6 \rangle}{\|\langle 2, 3 \rangle\| \|\langle 2, 6 \rangle\|} = \frac{11}{\sqrt{130}}$$

(b) Find the length of the curve from $t = 0$ to $t = t$.

**Solution:**

$$s(t) = \int_0^t \|r'(u)\| \, du = \int_0^t \sqrt{9u^4 + 4u^2} \, du$$

$$= \int_0^t u\sqrt{9u^2 + 4} \, du \quad \text{(use substitution, } w = 9u^2 + 4)$$

$$= \left. \frac{1}{27} (9u^2 + 4)^{3/2} \right|_0^t = \frac{1}{27} (9t^2 + 4)^{3/2} - \frac{8}{27}$$

(c) Find $\frac{ds}{dt}$ for your arc length function.

**Solution:**

$$s'(t) = \|r'(t)\| = \sqrt{9t^4 + 4t^2}$$
Lecture Problems

2. Let $z = f(x, y) = x^2 + 3y^2$.

(a) Draw a single graph (in the $xy$-plane) with the following slices: $z = -3, z = -2, z = -1, z = 0, z = 1, z = 2, z = 3$.

(b) Draw a single graph (in the $yz$-plane) with the following slices: $x = -3, x = -2, x = -1, x = 0, x = 1, x = 2, x = 3$.

(c) Draw a single graph (in the $xz$-plane) with the following slices: $y = -3, y = -2, y = -1, y = 0, y = 1, y = 2, y = 3$. 
Lecture Problems

2. Let \( z = f(x, y) = x^2 + 3y^2 \).

(a) Draw a single graph (in the \( xy \)-plane) with the following slices:
\[ z = -3, z = -2, z = -1, z = 0, z = 1, z = 2, z = 3. \]
Solution:

(b) Draw a single graph (in the \( yz \)-plane) with the following slices:
\[ x = -3, x = -2, x = -1, x = 0, x = 1, x = 2, x = 3. \]
Solution:
3. Draw the following graphs in $\mathbb{R}^3$. As many (or as few) slices in each direction that you need.

(a) $f(x, y) = 4 - 3x + 2y$
(b) $f(x, y) = \sqrt{100 - x^2 - y^2}$
(c) $f(x, y) = 3x - 7y$
(d) $f(x, y) = x^2 + xy$
(e) $f(x, y) = x/y$
(f) $4x^2 + y^2 = 16$
(g) $\frac{x}{4} = \frac{y^2}{4} + \frac{z^2}{9}$
(h) $z = x^2$
(i) $\frac{x^2}{9} + \frac{y^2}{12} + \frac{z^2}{9} = 1$
(j) $4x^2 - 3y^2 + 2z^2 = 0$
(k) $x^2 + y^2 - 2x = 0$