# Math 233 Warm Up Problems

**September 8, 2009**

1. Quiz

    (a) What is meant by "position vector?"
    (b) How can you tell the difference between a point and a vector?
    (c) Can you take the dot product of two points?
    (d) What do the following mean?

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad (1, 2, 3), \quad < 1, 2, 3 >$$

    (e) Which of the following make sense and which do not?

$$(A \cdot B) \times C$$
$$A \cdot (B \times C)$$

1. Quiz

(a) What is meant by "position vector?"
**Solution:** Given a point, we can consider it a vector as the vector from the origin to the point.

(b) How can you tell the difference between a point and a vector?
**Solution:** Algebraically they behave the same—you can add them, multiply by a constant, etc. But, geometrically we think of them differently. Vectors have direction and can move around in space (just so long as they are always pointing in the same way). Points can not move around space.

(c) Can you take the dot product of two points?
**Solution:** Sure, but we usually think of dot producting vectors. If you actually did this then you should probably consider this as the dot product of the position vectors (see earlier question).

(d) What do the following mean?

$$\begin{pmatrix}1\\2\\3\end{pmatrix}, \quad \begin{bmatrix}1\\2\\3\end{bmatrix}, \quad (1,2,3), \quad <1,2,3>$$

**Solution:** They can all represent points or vectors in $\mathbb{R}^3$.

(e) Which of the following make sense and which do not?

| | |
|---|---|
| $(A \cdot B) \times C$ | Makes no sense |
| $A \cdot (B \times C)$ | okay if $A, B, C$ are vectors |

2. Find a parametric equation for a circle centered at $(-11, 27)$ and with radius 93.
3. Find a parametric equation for the ellipse

$$30(x-5)^2 + 16(y+19)^2 = 36$$

2. Find a parametric equation for a circle centered at $(-11, 27)$ and with radius 93.

**Solution:**

$$r(t) = (-11 + 93\cos t, 27 + 93\sin t)$$

3. Find a parametric equation for the ellipse

$$30(x-5)^2 + 16(y+19)^2 = 36$$

**Solution:** It is probably easiest to recognize this as the ellipse:

$$\left(\frac{x-5}{\sqrt{6/5}}\right)^2 + \left(\frac{y+19}{3/2}\right)^2 = 1$$

and thus we can make

$$r(t) = \left(5 + \sqrt{6/5}\cos t, -19 + (3/2)\sin t\right)$$

(You can substitute these into the original equation and see that it works.)

## Lecture Problems

4. Set up integrals for the lengths of the curves

   (a) Length of $r(t) = (t, t^2, t^3)$ from $t = 1$ to $t = 7$.
   (b) Length of $r(t) = (e^t, e^{-t}, \ln t)$ from $t = 2$ to $t = 13$.
   (c) Length of $r(t) = (\cos t, \sin t, t, t^2)$ from $t = 0$ to $t = \pi$.

## Lecture Problems

4. Set up integrals for the lengths of the curves

   (a) Length of $r(t) = (t, t^2, t^3)$ from $t = 1$ to $t = 7$.
   **Solution:**

   $$\int_1^7 \sqrt{1 + (2t)^2 + (3t^2)^2}\, dt$$

   (b) Length of $r(t) = (e^t, e^{-t}, \ln t)$ from $t = 2$ to $t = 13$.
   **Solution:**

   $$\int_2^{13} \sqrt{(e^t)^2 + (-e^{-t})^2 + (1/t)^2}\, dt$$

   (c) Length of $r(t) = (\cos t, \sin t, t, t^2)$ from $t = 0$ to $t = \pi$.
   **Solution:**

   $$\int_0^{\pi} \sqrt{\sin^2 t + \cos^2 t + 1 + (2t)^2}\, dt$$