

Math 233 Warm Up Problems

September 4, 2009
Solutions

1. Consider a particle that moves along the curve in $\mathbb{R}^3$ given by

$$r(t) = (\cos t, \sin t, t)$$

- Find the position at time $t = \frac{\pi}{6}$.

Solution:

$$r(\pi/6) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\pi}{6} \right)$$

- Find the velocity at time $t = \frac{\pi}{6}$.

Solution:

$$r'(t) = (-\sin t, \cos t, 1)$$
$$r'(\pi/6) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 1 \right)$$

- Find the speed at time $t = \frac{\pi}{6}$.

Solution:

$$\text{Speed} = \|v(t)\|$$
$$\text{Speed}(\pi/6) = \left\| \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 1 \right) \right\| = \sqrt{2}$$

- Find the acceleration at time $t = \frac{\pi}{6}$.

Solution:

$$r''(t) = (-\cos t, -\sin t, 0)$$
$$r''(\pi/6) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0 \right)$$

- Find the equation of the line tangent to the curve at time $t = \frac{\pi}{6}$.

Solution:

$$L(t) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\pi}{6} \right) + t \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 1 \right)$$

- Find the equation of the plane through the curve at $r(\pi/6)$ and normal to the curve.

Solution: We have a point and a normal direction:

$$-\frac{1}{2} \left(x - \frac{\sqrt{3}}{2} \right) + \frac{\sqrt{3}}{2} \left(y - \frac{1}{2} \right) + \left(z - \frac{\pi}{6} \right) = 0$$

$$-\frac{1}{2}x + \frac{\sqrt{3}}{2}y + z = \frac{\pi}{6}$$

Lecture Problems

2. Find a parametric equation for a circle centered at $(9, -2)$ and with radius 19.

Solution:

$$r(t) = (9 + 19 \cos t, -2 + 19 \sin t)$$

3. Find a parametric equation for the ellipse

$$9(x - 7)^2 + 16(y + \pi)^2 = 36$$