Math 233 Warm Up Problems
September 4, 2009
1. Consider a particle that moves along the curve in $\mathbb{R}^3$ given by

$$r(t) = (\cos t, \sin t, t)$$

- Find the position at time $t = \frac{\pi}{6}$.
- Find the velocity at time $t = \frac{\pi}{6}$.
- Find the speed at time $t = \frac{\pi}{6}$.
1. Consider a particle that moves along the curve in $\mathbb{R}^3$ given by

$$r(t) = (\cos t, \sin t, t)$$

- Find the position at time $t = \frac{\pi}{6}$.
  **Solution:**
  $$r\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\pi}{6}\right)$$

- Find the velocity at time $t = \frac{\pi}{6}$.
  **Solution:**
  $$r'(t) = (-\sin t, \cos t, 1)$$
  $$r'\left(\frac{\pi}{6}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 1\right)$$

- Find the speed at time $t = \frac{\pi}{6}$.
  **Solution:**
  $$\text{Speed} = ||v(t)||$$
  $$\text{Speed}\left(\frac{\pi}{6}\right) = \left\|\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 1\right)\right\| = \sqrt{2}$$
Find the acceleration at time $t = \frac{\pi}{6}$.

Find the equation of the line tangent to the curve at time $t = \frac{\pi}{6}$.

Find the equation of the plane through the curve at $r(\pi/6)$ and normal to the curve.
Find the acceleration at time \( t = \frac{\pi}{6} \).

**Solution:**

\[
r''(t) = (-\cos t, -\sin t, 0)
\]

\[
r''\left(\frac{\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right)
\]

Find the equation of the line tangent to the curve at time \( t = \frac{\pi}{6} \).

**Solution:**

\[
L(t) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\pi}{6}\right) + t \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 1\right)
\]

Find the equation of the plane through the curve at \( r(\pi/6) \) and normal to the curve.

**Solution:** We have a point and a normal direction:

\[
-\frac{1}{2} \left(x - \frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} \left(y - \frac{1}{2}\right) + \left(z - \frac{\pi}{6}\right) = 0
\]

\[
-\frac{1}{2}x + \frac{\sqrt{3}}{2}y + z = \frac{\pi}{6}
\]
Lecture Problems

2. Find a parametric equation for a circle centered at \((9, -2)\) and with radius 19.

3. Find a parametric equation for the ellipse

\[ 9(x - 7)^2 + 16(y + \pi)^2 = 36 \]
Lecture Problems

2. Find a parametric equation for a circle centered at \((9, -2)\) and with radius 19.

Solution:

\[ r(t) = (9 + 19 \cos t, -2 + 19 \sin t) \]

3. Find a parametric equation for the ellipse

\[ 9(x - 7)^2 + 16(y + \pi)^2 = 36 \]