

Math 233 Warm Up Problems

September 2, 2009

Solutions

1. Find the angle between the two planes

$$2x + y - 5z = 32$$

$$x + 2y + 7z = \sqrt{17}$$

Solution: The angle between the planes is found by finding the angle between the normal vectors of the planes:

$$\begin{aligned}\cos \theta &= \frac{N_1 \cdot N_2}{\|N_1\| \|N_2\|} = \frac{(2, 1, -5) \cdot (1, 2, 7)}{\|(2, 1, -5)\| \|(1, 2, 7)\|} \\ &= -\frac{31}{18\sqrt{5}} \approx -0.7702 \\ \theta &\approx 2.45\end{aligned}$$

Lecture Problems

2. Calculate the determinants

$$\begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix} = -10$$

$$\begin{vmatrix} 1 & 2 & 0 \\ 4 & -2 & 1 \\ 0 & 1 & 5 \end{vmatrix} = -51$$

$$\begin{vmatrix} -5 & -78 & 31 \\ 0 & 2 & 891 \\ 0 & 0 & -3 \end{vmatrix} = 30$$

3. Calculate the cross product

$$(-2, 1, -2) \times (1, -1, 3) = (1, 4, 1)$$

4. Find two different unit vectors both orthogonal to $(-2, 1, -2)$ and $(1, -1, 3)$.

Solution: Use the previous result.

$$\left(\frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}} \right), \quad \left(-\frac{1}{\sqrt{18}}, -\frac{4}{\sqrt{18}}, -\frac{1}{\sqrt{18}} \right),$$

5. Find a parametrization of the circle centered at $(1, 5)$ with radius 7.

Solution:

$$r(t) = (1 + 7 \cos t, 5 + 7 \sin t)$$

6. Find the equation of the line tangent to the curve

$$r(t) = (-2t + 1, t^2, t^3)$$

when $t = 1$.

Solution:

$$r(1) = (-1, 1, 1)$$

$$r'(t) = (-2, 2t, 3t^2)$$

$$r'(1) = (-2, 2, 3)$$

$$L(t) = (-1, 1, 1) + t(-2, 2, 3)$$