Math 233 Warm Up Problems

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1. Find the angle between the two planes

$$2x + y - 5z = 32$$
$$x + 2y + 7z = \sqrt{17}$$

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Solution: The angle between the planes is found by finding the angle between the normal vectors of the planes:

$$\begin{aligned}\cos\theta &= \frac{N_1 \cdot N_2}{\|N_1\|\|N_2\|} = \frac{(2,1,-5)\cdot(1,2,7)}{\|(2,1,-5)\|\|(1,2,7)\|} \\ &= -\frac{31}{18\sqrt{5}} \approx -0.7702 \\ \theta &\approx 2.45\end{aligned}$$

Lecture Problems

2. Calculate the determinants

$$\begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 2 & 0 \\ 4 & -2 & 1 \\ 0 & 1 & 5 \end{vmatrix} =$$

$$\begin{vmatrix} -5 & -78 & 31 \\ 0 & 2 & 891 \\ 0 & 0 & -3 \end{vmatrix} =$$

Lecture Problems

2. Calculate the determinants

$$\begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix} = -10$$

$$\begin{vmatrix} 1 & 2 & 0 \\ 4 & -2 & 1 \\ 0 & 1 & 5 \end{vmatrix} = -51$$

$$\begin{vmatrix} -5 & -78 & 31 \\ 0 & 2 & 891 \\ 0 & 0 & -3 \end{vmatrix} = 30$$

3. Calculate the cross product

$$(-2, 1, -2) \times (1, -1, 3) =$$

4. Find two different unit vectors both orthogonal to $(-2, 1, -2)$ and $(1, -1, 3)$.

3. Calculate the cross product

$$(-2, 1, -2) \times (1, -1, 3) = (1, 4, 1)$$

4. Find two different unit vectors both orthogonal to $(-2, 1, -2)$ and $(1, -1, 3)$.
Solution: Use the previous result.

$$\left(\frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right), \quad \left(-\frac{1}{\sqrt{18}}, -\frac{4}{\sqrt{18}}, -\frac{1}{\sqrt{18}}\right),$$

5. Find a parametrization of the circle centered at $(1, 5)$ with radius 7.
6. Find the equation of the line tangent to the curve

$$r(t) = (-2t + 1, t^2, t^3)$$

when $t = 1$.

5. Find a parametrization of the circle centered at $(1, 5)$ with radius 7.
   Solution:

$$r(t) = (1 + 7\cos t, 5 + 7\sin t)$$

6. Find the equation of the line tangent to the curve

$$r(t) = (-2t + 1, t^2, t^3)$$

   when $t = 1$.
   Solution:

$$\begin{aligned} r(1) &= (-1, 1, 1) \\ r'(t) &= (-2, 2t, 3t^2) \\ r'(1) &= (-2, 2, 3) \\ L(t) &= (-1, 1, 1) + t(-2, 2, 3) \end{aligned}$$