Math 233 Warm Up Problems

September 1, 2009

Solutions

1. Determine where, if anywhere, the two lines intersect

$$\begin{aligned}\mathbf{r_1}(t) =&(0,2,9) + t(3,4,-1)\\ \mathbf{r_2}(t) =&(6,4,5) + t(0,6,2)\end{aligned}$$

Solution: The lines intersect at the point $(6, 10, 7)$. It is important to change the parameter in one of the lines:

$$\begin{aligned}\mathbf{r_1}(t) =&(0,2,9) + t(3,4,-1)\\ \mathbf{r_2}(s) =&(6,4,5) + s(0,6,2)\end{aligned}$$

and then you can solve systems of equations $r_1(t) = r_2(s)$ and find that $t = 2$ and $s = 1$ gives a solution.

Lecture Problems

2. Find the equation of the plane that goes through the points

$$p_1 = (1,2,3), \quad p_2 = (1,3,2), \quad p_3 = (-1,3,10)$$

Solution: Note that the plane is parallel to the vectors $v_1 = p_2 - p_1 = (0, 1, -1)$ and $v_2 = p_3 - p_1 = (-2, 1, 7)$. Thus, we need to find $N = (a, b, c)$ so that $N \cdot v_1 = 0$ and $N \cdot v_2 = 0$ giving equations

$$\begin{aligned}b - c =&0\\ -2a + b + 7c =&0\end{aligned}$$

A possible solution is $N = (4, 1, 1)$ giving the equation

$$4x + y + z = 9$$

3. Find the distance between the point $P = (2, 10, 1)$ and the plane $x + y - 2z = 5$.

Solution: The normal vector is $N = (1, 1, -2)$. I used $Q = (5, 0, 0)$ as my point on the plane. Then $v = \overrightarrow{QP} = (-3, 10, 1)$ and the distance is

$$\begin{aligned}\|\text{Projection of } \overrightarrow{QP} \text{ to } N\| =& \left\| \frac{\overrightarrow{QP} \cdot N}{N \cdot N} N \right\|\\ =&\left\| \frac{5}{6} N \right\|\\ =&\left\| \left(\frac{5}{6}, \frac{5}{6}, -\frac{5}{3} \right) \right\|\\ =&\frac{5}{\sqrt{6}}\end{aligned}$$

4. Calculate the cross product

$$(1,2,5) \times (-1,2,-1) = (-12,-4,4)$$

5. Find two different unit vectors both orthogonal to $(1,2,5)$ and $(-1,2,-1)$.

   Solution: Use the previous result.

$$\left(-\frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right), \quad \left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}\right)$$