Math 233 Warm Up Problems  
September 1, 2009  
Solutions

1. Determine where, if anywhere, the two lines intersect

\[ r_1(t) = (0, 2, 9) + t(3, 4, -1) \]
\[ r_2(t) = (6, 4, 5) + t(0, 6, 2) \]

**Solution:** The lines intersect at the point (6, 10, 7). It is important to change the parameter in one of the lines:

\[ r_1(t) = (0, 2, 9) + t(3, 4, -1) \]
\[ r_2(s) = (6, 4, 5) + s(0, 6, 2) \]

and then you can solve systems of equations \( r_1(t) = r_2(s) \) and find that \( t = 2 \) and \( s = 1 \) gives a solution.

**Lecture Problems**

2. Find the equation of the plane that goes through the points

\[ p_1 = (1, 2, 3), \quad p_2 = (1, 3, 2), \quad p_3 = (-1, 3, 10) \]

**Solution:** Note that the plane is parallel to the vectors \( v_1 = p_2 - p_1 = (0, 1, -1) \) and \( v_2 = p_3 - p_1 = (-2, 1, 7) \). Thus, we need to find \( N = (a, b, c) \) so that \( N \cdot v_1 = 0 \) and \( N \cdot v_2 = 0 \) giving equations

\[ b - c = 0 \]
\[ -2a + b + 7c = 0 \]

A possible solution is \( N = (4, 1, 1) \) giving the equation

\[ 4x + y + z = 9 \]

3. Find the distance between the point \( P = (2, 10, 1) \) and the plane \( x + y - 2z = 5 \).

**Solution:** The normal vector is \( N = (1, 1, -2) \). I used \( Q = (5, 0, 0) \) as my point on the plane. Then \( v = \overrightarrow{QP} = (-3, 10, 1) \) and the distance is

\[
\|\text{Projection of } \overrightarrow{QP} \text{ to } N\| = \left\| \frac{\overrightarrow{QP} \cdot N}{N \cdot N} \right\|
\]

\[ = \left\| \frac{5}{6} N \right\|
\]

\[ = \left\| \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix} \right\|
\]

\[ = \frac{5}{\sqrt{6}} \]
4. Calculate the cross product

\[(1, 2, 5) \times (-1, 2, -1) = (-12, -4, 4)\]

5. Find two different unit vectors both orthogonal to \((1, 2, 5)\) and \((-1, 2, -1)\).

**Solution:** Use the previous result.

\[\left(-\frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right), \left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}\right)\]