Math 233 Warm Up Problems
September 1, 2009
1. Determine where, if anywhere, the two lines intersect

\[ r_1(t) = (0, 2, 9) + t(3, 4, -1) \]
\[ r_2(t) = (6, 4, 5) + t(0, 6, 2) \]
1. Determine where, if anywhere, the two lines intersect

\[ \mathbf{r}_1(t) = (0, 2, 9) + t(3, 4, -1) \]
\[ \mathbf{r}_2(t) = (6, 4, 5) + t(0, 6, 2) \]

**Solution:** The lines intersect at the point \((6, 10, 7)\). It is important to change the parameter in one of the lines:

\[ \mathbf{r}_1(t) = (0, 2, 9) + t(3, 4, -1) \]
\[ \mathbf{r}_2(s) = (6, 4, 5) + s(0, 6, 2) \]

and then you can solve systems of equations \( \mathbf{r}_1(t) = \mathbf{r}_2(s) \) and find that \( t = 2 \) and \( s = 1 \) gives a solution.
Lecture Problems

2. Find the equation of the plane that goes through the points

\[ p_1 = (1, 2, 3), \quad p_2 = (1, 3, 2), \quad p_3 = (-1, 3, 10) \]
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**Solution:** Note that the plane is parallel to the vectors
\[ v_1 = p_2 - p_1 = (0, 1, -1) \] and \[ v_2 = p_3 - p_1 = (-2, 1, 7) \]. Thus, we need to find \( N = (a, b, c) \) so that \( N \cdot v_1 = 0 \) and \( N \cdot v_2 = 0 \) giving equations

\[
\begin{align*}
    b - c &= 0 \\
    -2a + b + 7c &= 0
\end{align*}
\]

A possible solution is \( N = (4, 1, 1) \) giving the equation

\[ 4x + y + z = 9 \]
3. Find the distance between the point $P = (2, 10, 1)$ and the plane $x + y - 2z = 5$. 
3. Find the distance between the point \( P = (2, 10, 1) \) and the plane \( x + y - 2z = 5 \).

**Solution:** The normal vector is \( N = (1, 1, -2) \). I used \( Q = (5, 0, 0) \) as my point on the plane. Then \( v = Q\vec{P} = (-3, 10, 1) \) and the distance is

\[
\|\text{Projection of } Q\vec{P} \text{ to } N\| = \left\| \frac{Q\vec{P} \cdot N}{N \cdot N} N \right\|
\]

\[
= \left\| \frac{5}{6} N \right\|
\]

\[
= \left\| \left( \frac{5}{6}, \frac{5}{6}, -\frac{5}{3} \right) \right\|
\]

\[
= \frac{5}{\sqrt{6}}
\]
4. Calculate the cross product

\[(1, 2, 5) \times (-1, 2, -1) =\]

5. Find two different unit vectors both orthogonal to \((1, 2, 5)\) and \((-1, 2, -1)\).
4. Calculate the cross product

\((1, 2, 5) \times (-1, 2, -1) = (-12, -4, 4)\)

5. Find two different unit vectors both orthogonal to \((1, 2, 5)\) and \((-1, 2, -1)\).

**Solution:** Use the previous result.

\[
\left( -\frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right), \quad \left( \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}} \right)
\]