Math 233 - October , 2009

- Setting up and calculating double integrals.

For each of the problems, set up the integral in both possible orders of integration ($dy\,dx$ and $dx\,dy$). Calculate the integral both ways (and make sure you get the same answer).

1. Let R be the rectangle $[1,2] \times [3,10]$.

$$\iint_R x + 2y \; dA =$$

1. Let R be the rectangle $[1,2] \times [3,10]$.

$$\begin{aligned}\iint_R x + 2y \; dA &= \int_1^2 \int_3^{10} \; dy \; dx \\ &= \int_3^{10} \int_1^2 \; dx \; dy \\ &= \frac{203}{2}\end{aligned}$$

Lecture Problems

2. For the following problems, draw the region of integration and change the order of integration.

   (a)

   $$\int_0^1 \int_0^x f(x,y)\, dy\, dx =$$

   (b)

   $$\int_0^1 \int_{x^2}^{x^{1/4}} f(x,y)\, dy\, dx =$$

   (c)

   $$\int_0^1 \int_{-y}^{y} f(x,y)\, dx\, dy =$$

Lecture Problems

2. For the following problems, draw the region of integration and change the order of integration.

   (a)

$$\int_0^1 \int_0^x f(x,y)\,dy\,dx = \int_0^1 \int_y^1 f(x,y)\,dx\,dy$$

   (b)

$$\int_0^1 \int_{x^2}^{x^{1/4}} f(x,y)\,dy\,dx = \int_0^1 \int_{y^4}^{\sqrt{y}} f(x,y)\,dx\,dy$$

   (c)

$$\int_0^1 \int_{-y}^{y} f(x,y)\,dx\,dy = \int_{-1}^0 \int_{-x}^1 f(x,y)\,dy\,dx + \int_0^1 \int_x^1 f(x,y)\,dy\,dx$$

3. Let R be the region in the first quadrant of the xy-plane that is bounded by a quarter of the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $y = 0$.
   Note: You may need a trig substitution to actually compute the integral.

$$\iint_R x^2 + y^2 \, dA =$$

4. Let R be the region between the the curves $y = x^2$ and $y = 1$.

$$\iint_R xy \, dA =$$

3. Let R be the region in the first quadrant of the xy-plane that is bounded by a quarter of the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $y = 0$.
   Note: You may need a trig substitution to actually compute the integral.

$$\iint_R x^2 + y^2 \, dA = 2\pi$$

4. Let R be the region between the the curves $y = x^2$ and $y = 1$.

$$\iint_R xy \, dA = 0$$

5. Compute

$$\int_0^4 \int_{x/2}^2 e^{y^2}\, dy\, dx =$$

6. Let R be the triangular region with vertices $(0,0)$, $(0,4)$, and $(1,4)$.

$$\iint_R x + y\, dA =$$

7. Let R be region between $y = x^2$ and $y = \sqrt{x}$.

$$\iint_R x^2 + 2y\, dA =$$

8. Let R be region between the graphs $y = x$ and $y = x^3$. Note: R has two parts.

$$\iint_R x\, dA =$$

5. Compute

$$\int_0^4 \int_{x/2}^2 e^{y^2} \, dy \, dx = e^4 - 1$$

Note: you have to change the order of integration to compute this.

6. Let R be the triangular region with vertices $(0,0)$, $(0,4)$, and $(1,4)$.

$$\iint_R x + y \, dA = 6$$

7. Let R be region between $y = x^2$ and $y = \sqrt{x}$.

$$\iint_R x^2 + 2y \, dA = \frac{27}{70}$$

8. Let R be region between the graphs $y = x$ and $y = x^3$. Note: R has two parts.

$$\iint_R x \, dA = \frac{2}{15} - \frac{2}{15} = 0$$

9. Find the volume of the tetrahedron bounded by the coordinate planes and $3x + 6y + 4z = 12$.
10. Find the volume of the tetrahedron bounded by the coordinate planes and $z = 6 - 2x - 3y$.
11. Find the volume of the wedge bounded by the coordinate planes and the planes $x = 5$ and $y + 2z - 4 = 0$.
12. Find the volume of the solid in the first octant bounded by the surface $9x^2 + 4y^2 = 36$ and the plane $9x + 4y - 6z = 0$.

9. Find the volume of the tetrahedron bounded by the coordinate planes and $3x + 6y + 4z = 12$.
   Solution: 4
10. Find the volume of the tetrahedron bounded by the coordinate planes and $z = 6 - 2x - 3y$.
   Solution: 6
11. Find the volume of the wedge bounded by the coordinate planes and the planes $x = 5$ and $y + 2z - 4 = 0$.
   Solution: 20
12. Find the volume of the solid in the first octant bounded by the surface $9x^2 + 4y^2 = 36$ and the plane $9x + 4y - 6z = 0$.
   Solution: 10