Setting up and calculating double integrals.

For each of the problems, set up the integral in both possible orders of integration \((dy \, dx)\) and \((dx \, dy)\). Calculate the integral both ways (and make sure you get the same answer).
1. Let $R$ be the rectangle $[1, 2] \times [3, 10]$.

$$\int \int_R x + 2y \ dA =$$
1. Let $R$ be the rectangle $[1, 2] \times [3, 10]$.

\[
\int \int_{R} (x + 2y) \, dA = \int_{1}^{2} \int_{3}^{10} dy \, dx \\
= \int_{3}^{10} \int_{1}^{2} dx \, dy \\
= \frac{203}{2}
\]
2. For the following problems, draw the region of integration and change the order of integration.

(a) \[ \int_0^1 \int_0^x f(x, y) \, dy \, dx = \]

(b) \[ \int_0^1 \int_{x^2}^{x^{1/4}} f(x, y) \, dy \, dx = \]

(c) \[ \int_0^1 \int_{-y}^{y} f(x, y) \, dx \, dy = \]
Lecture Problems

2. For the following problems, draw the region of integration and change the order of integration.

(a)

\[ \int_0^1 \int_0^x f(x, y) \, dy \, dx = \int_0^1 \int_y^1 f(x, y) \, dx \, dy \]

(b)

\[ \int_0^1 \int_{x^2}^{x^{1/4}} f(x, y) \, dy \, dx = \int_0^1 \int_{y^4}^\sqrt{y} f(x, y) \, dx \, dy \]

(c)

\[ \int_0^1 \int_{-y}^y f(x, y) \, dx \, dy = \int_{-1}^0 \int_{-x}^1 f(x, y) \, dy \, dx + \int_0^1 \int_x^1 f(x, y) \, dy \, dx \]
3. Let $R$ be the region in the first quadrant of the $xy$-plane that is bounded by a quarter of the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $y = 0$. Note: You may need a trig substitution to actually compute the integral.

$$\int\int_R x^2 + y^2 \, dA =$$

4. Let $R$ be the region between the the curves $y = x^2$ and $y = 1$.

$$\int\int_R xy \, dA =$$
3. Let \( R \) be the region in the first quadrant of the \( xy \)-plane that is bounded by a quarter of the circle \( x^2 + y^2 = 4 \) and the lines \( x = 0 \) and \( y = 0 \).
   Note: You may need a trig substitution to actually compute the integral.
   \[
   \iint_R x^2 + y^2 \, dA = 2\pi
   \]

4. Let \( R \) be the region between the the curves \( y = x^2 \) and \( y = 1 \).
   \[
   \iint_R xy \, dA = 0
   \]
5. Compute
\[ \int_0^4 \int_{x/2}^2 e^{y^2} \, dy \, dx = \]

6. Let \( R \) be the triangular region with vertices (0, 0), (0, 4), and (1, 4).
\[ \int \int_R x + y \, dA = \]

7. Let \( R \) be region between \( y = x^2 \) and \( y = \sqrt{x} \).
\[ \int \int_R x^2 + 2y \, dA = \]

8. Let \( R \) be region between the graphs \( y = x \) and \( y = x^3 \). Note: \( R \) has two parts.
\[ \int \int_R x \, dA = \]
5. Compute
\[ \int_0^4 \int_{x/2}^2 e^{y^2} \, dy \, dx = e^4 - 1 \]

Note: you have to change the order of integration to compute this.

6. Let \( R \) be the triangular region with vertices \((0, 0), (0, 4), \) and \((1, 4)\).
\[ \iint_R x + y \, dA = 6 \]

7. Let \( R \) be region between \( y = x^2 \) and \( y = \sqrt{x} \).
\[ \iint_R x^2 + 2y \, dA = \frac{27}{70} \]

8. Let \( R \) be region between the graphs \( y = x \) and \( y = x^3 \). Note: \( R \) has two parts.
\[ \iint_R x \, dA = \frac{2}{15} - \frac{2}{15} = 0 \]
9. Find the volume of the tetrahedron bounded by the coordinate planes and $3x + 6y + 4z = 12$.

10. Find the volume of the tetrahedron bounded by the coordinate planes and $z = 6 - 2x - 3y$.

11. Find the volume of the wedge bounded by the coordinate planes and the planes $x = 5$ and $y + 2z - 4 = 0$.

12. Find the volume of the solid in the first octant bounded by the surface $9x^2 + 4y^2 = 36$ and the plane $9x + 4y - 6z = 0$. 
9. Find the volume of the tetrahedron bounded by the coordinate planes and $3x + 6y + 4z = 12$.
   **Solution:** 4

10. Find the volume of the tetrahedron bounded by the coordinate planes and $z = 6 - 2x - 3y$.
    **Solution:** 6

11. Find the volume of the wedge bounded by the coordinate planes and the planes $x = 5$ and $y + 2z - 4 = 0$.
    **Solution:** 20

12. Find the volume of the solid in the first octant bounded by the surface $9x^2 + 4y^2 = 36$ and the plane $9x + 4y - 6z = 0$.
    **Solution:** 10