Math 233 - October 29, 2009
Solutions

- Computing double integrals.

1. Compute the integrals by viewing the integral as a volume.

   (a) Let R be the rectangle $[0, 3] \times [0, 4]$. Let

   $$f(x, y) = \begin{cases} -5 & \text{if } y \geq 3 \\ 2 & \text{if } y < 3 \end{cases}$$

   $$\iint_R f(x, y)\, dy\, dx = (2)(8) + (-5)(3) = 1$$

   (b) Let R be the rectangle $[0, 3] \times [0, 4]$. Let

   $$f(x, y) = \begin{cases} 3 & \text{if } y \leq 3 \text{ and } x \leq 1 \\ 2 & \text{if } y \leq 3 \text{ and } x > 1 \\ 6 & \text{if } y > 3 \text{ and } x \leq 1 \\ -5 & \text{if } y > 3 \text{ and } x > 1 \end{cases}$$

   $$\iint_R f(x, y)\, dy\, dx = (3)(3) + (6)(2) + (1)(6) + (3)(-5) = 12$$

Lecture Problems

2. Compute the iterated (repeated) integrals

   (a)

   $$\int_{-1}^{1} \int_{0}^{1} x^2 y + 1\, dy\, dx = \int_{-1}^{1} \frac{x^2 + 2}{2}\, dx = \frac{7}{3}$$

   (b)

   $$\int_{0}^{1} \int_{-1}^{1} x^2 y + 1\, dx\, dy = \int_{0}^{1} \frac{2(y + 3)}{3}\, dy = \frac{7}{3}$$

   (c)

   $$\int_{0}^{\pi/2} \int_{0}^{\pi/4} \sin(x + y)\, dy\, dx = \int_{0}^{\pi/2} \cos x - \cos(x + \pi/4)\, dx = 1$$

3. Draw the regions of integration, determine limits for an integral over the region.

(a) Let R be the triangle with vertices $(0,0)$, $(1,1)$ and $(1,0)$.

$$\iint_R f(x,y)\,dA = \int_0^1 \int_0^x f(x,y)\,dy\,dx = \int_0^1 \int_y^1 f(x,y)\,dx\,dy$$

(b) Let R be the triangle with vertices $(0,0)$, $(1,1)$ and $(0,1)$.

$$\iint_R f(x,y)\,dA = \int_0^1 \int_x^1 f(x,y)\,dy\,dx = \int_0^1 \int_0^1 f(x,y)\,dx\,dy$$

(c) Let R be the disc, $x^2+y^2 \le 4$.

$$\iint_R f(x,y)\,dA = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\,dy\,dx = \int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x,y)\,dx\,dy$$

(d) Let R be the upper half-disc, $y \ge 0$, $x^2+y^2 \le 1$.

$$\iint_R f(x,y)\,dA = \int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x,y)\,dy\,dx = \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

(e) Let R be the region above $y=1$ and below $y=2-x^2$.

$$\iint_R f(x,y)\,dA = \int_{-1}^1 \int_1^{2-x^2} f(x,y)\,dy\,dx = \int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x,y)\,dx\,dy$$

4. Let $f(x,y) = x+2y$ and compute $\iint_R f(x,y)\,dA$ for the regions in the previous problem.

(a) Let R be the triangle with vertices $(0,0)$, $(1,1)$ and $(1,0)$.

$$\iint_R x+2y\,dA = \frac{2}{3}$$

(b) Let R be the triangle with vertices $(0,0)$, $(1,1)$ and $(0,1)$.

$$\iint_R x+2y\,dA = \frac{5}{6}$$

(c) Let R be the disc, $x^2+y^2 \le 4$.

$$\iint_R x+2y\,dA = 0$$

(d) Let R be the upper half-disc, $y \ge 0$, $x^2+y^2 \le 1$.

$$\iint_R x+2y\,dA = \frac{4}{3}$$

(e) Let R be the region above $y=1$ and below $y=2-x^2$.

$$\iint_R x+2y\,dA = \frac{56}{15}$$