

Math 233 - October 29, 2009

- Computing double integrals.

1. Compute the integrals by viewing the integral as a volume.

(a) Let R be the rectangle $[0, 3] \times [0, 4]$. Let

$$f(x, y) = \begin{cases} -5 & \text{if } y \geq 3 \\ 2 & \text{if } y < 3 \end{cases}$$

$$\iint_R f(x, y) \, dy \, dx =$$

(b) Let R be the rectangle $[0, 3] \times [0, 4]$. Let

$$f(x, y) = \begin{cases} 3 & \text{if } y \leq 3 \text{ and } x \leq 1 \\ 2 & \text{if } y \leq 3 \text{ and } x > 1 \\ 6 & \text{if } y > 3 \text{ and } x \leq 1 \\ -5 & \text{if } y > 3 \text{ and } x > 1 \end{cases}$$

$$\iint_R f(x, y) \, dy \, dx =$$

1. Compute the integrals by viewing the integral as a volume.

(a) Let R be the rectangle $[0, 3] \times [0, 4]$. Let

$$f(x, y) = \begin{cases} -5 & \text{if } y \geq 3 \\ 2 & \text{if } y < 3 \end{cases}$$

$$\iint_R f(x, y) \, dy \, dx = (2)(8) + (-5)(3) = 1$$

(b) Let R be the rectangle $[0, 3] \times [0, 4]$. Let

$$f(x, y) = \begin{cases} 3 & \text{if } y \leq 3 \text{ and } x \leq 1 \\ 2 & \text{if } y \leq 3 \text{ and } x > 1 \\ 6 & \text{if } y > 3 \text{ and } x \leq 1 \\ -5 & \text{if } y > 3 \text{ and } x > 1 \end{cases}$$

$$\iint_R f(x, y) \, dy \, dx = (3)(3) + (6)(2) + (1)(6) + (3)(-5) = 12$$

Lecture Problems

2. Compute the iterated (repeated) integrals

(a)

$$\int_{-1}^1 \int_0^1 x^2 y + 1 \, dy \, dx =$$

(b)

$$\int_0^1 \int_{-1}^1 x^2 y + 1 \, dx \, dy =$$

(c)

$$\int_0^{\pi/2} \int_0^{\pi/4} \sin(x + y) \, dy \, dx =$$

Lecture Problems

2. Compute the iterated (repeated) integrals

(a)

$$\int_{-1}^1 \int_0^1 x^2 y + 1 \, dy \, dx = \int_{-1}^1 \frac{x^2 + 2}{2} \, dx = \frac{7}{3}$$

(b)

$$\int_0^1 \int_{-1}^1 x^2 y + 1 \, dx \, dy = \int_0^1 \frac{2(y + 3)}{3} \, dy = \frac{7}{3}$$

(c)

$$\int_0^{\pi/2} \int_0^{\pi/4} \sin(x+y) \, dy \, dx = \int_0^{\pi/2} \cos x - \cos(x + \pi/4) \, dx = 1$$

3. Draw the regions of integration, determine limits for an integral over the region.

(a) Let R be the triangle with vertices $(0,0)$, $(1,1)$ and $(1,0)$.

$$\iint_R f(x,y) \, dA =$$

(b) Let R be the triangle with vertices $(0,0)$, $(1,1)$ and $(0,1)$.

$$\iint_R f(x,y) \, dA =$$

(c) Let R be the disc, $x^2 + y^2 \leq 4$.

$$\iint_R f(x,y) \, dA =$$

(d) Let R be the upper half-disc, $y \geq 0$, $x^2 + y^2 \leq 1$.

$$\iint_R f(x,y) \, dA =$$

(e) Let R be the region above $y = 1$ and below $y = 2 - x^2$.

$$\iint_R f(x,y) \, dA =$$

3. Draw the regions of integration, determine limits for an integral over the region.

(a) Let R be the triangle with vertices $(0, 0)$, $(1, 1)$ and $(1, 0)$.

$$\iint_R f(x, y) \, dA = \int_0^1 \int_0^x f(x, y) \, dy \, dx = \int_0^1 \int_y^1 f(x, y) \, dx \, dy$$

(b) Let R be the triangle with vertices $(0, 0)$, $(1, 1)$ and $(0, 1)$.

$$\iint_R f(x, y) \, dA = \int_0^1 \int_x^1 f(x, y) \, dy \, dx = \int_0^1 \int_0^1 f(x, y) \, dx \, dy$$

(c) Let R be the disc, $x^2 + y^2 \leq 4$.

$$\iint_R f(x, y) \, dA = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y) \, dy \, dx = \int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x, y) \, dx \, dy$$

(d) Let R be the upper half-disc, $y \geq 0$, $x^2 + y^2 \leq 1$.

$$\iint_R f(x, y) \, dA = \int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x, y) \, dy \, dx = \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) \, dx \, dy$$

(e) Let R be the region above $y = 1$ and below $y = 2 - x^2$.

$$\iint_R f(x, y) \, dA = \int_{-1}^1 \int_1^{2-x^2} f(x, y) \, dy \, dx = \int_{-1}^1 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x, y) \, dx \, dy$$

4. Let $f(x, y) = x + 2y$ and compute $\iint_R f(x, y) dA$ for the regions in the previous problem.

(a) Let R be the triangle with vertices $(0, 0)$, $(1, 1)$ and $(1, 0)$.

$$\iint_R x + 2y dA =$$

(b) Let R be the triangle with vertices $(0, 0)$, $(1, 1)$ and $(0, 1)$.

$$\iint_R x + 2y dA =$$

(c) Let R be the disc, $x^2 + y^2 \leq 4$.

$$\iint_R x + 2y dA =$$

(d) Let R be the upper half-disc, $y \geq 0$, $x^2 + y^2 \leq 1$.

$$\iint_R x + 2y dA =$$

(e) Let R be the region above $y = 1$ and below $y = 2 - x^2$.

$$\iint_R x + 2y dA =$$

4. Let $f(x, y) = x + 2y$ and compute $\iint_R f(x, y) dA$ for the regions in the previous problem.

(a) Let R be the triangle with vertices $(0, 0)$, $(1, 1)$ and $(1, 0)$.

$$\iint_R x + 2y dA = \frac{2}{3}$$

(b) Let R be the triangle with vertices $(0, 0)$, $(1, 1)$ and $(0, 1)$.

$$\iint_R x + 2y dA = \frac{5}{6}$$

(c) Let R be the disc, $x^2 + y^2 \leq 4$.

$$\iint_R x + 2y dA = 0$$

(d) Let R be the upper half-disc, $y \geq 0$, $x^2 + y^2 \leq 1$.

$$\iint_R x + 2y dA = \frac{4}{3}$$

(e) Let R be the region above $y = 1$ and below $y = 2 - x^2$.

$$\iint_R x + 2y dA = \frac{56}{15}$$