Math 233 - October 27, 2009
Solutions

• If a vector field satisfies the mixed partial condition, \( \text{curl } F = (0, 0, 0) \), how can we tell if \( F \) is conservative on its entire domain?

• How do we integrate in \( \mathbb{R}^2 \)?

1. Let \( F = \left( \frac{y^2 - 3y + x^2}{y^2 + x^2}, -\frac{y^2 + x^2 - 3x}{y^2 + x^2} \right) \)

   (a) With \( F = (f, g) \), find \( f_y \) and \( g_x \). Is \( f_y = g_x \)?

   **Solution:**
   
   \[ f_y = g_x = \frac{3(y - x)(y + x)}{(y^2 + x^2)^2} \]

   (b) Let \( r(t) = (\cos t, \sin t), \; 0 \leq t \leq 2\pi \). Compute
   
   \[ \frac{1}{2\pi} \oint_r F \cdot dr = 3 \]

   (c) Find, if possible, \( k \) and \( \phi \) so that \( F = kG + \nabla \phi \) where \( G = (-y/(x^2 + y^2), x/(x^2 + y^2)) \).

   \[ k = 3 \]
   \[ \nabla \phi = (1, -1) \]
   \[ \phi = x - y \]

**Lecture Problems**

2. Let \( R \) be the rectangle \([1, 3] \times [2, 8]\). Let

   \[ f(x, y) = 7 \]

   \[ \iint_R f(x, y) \, dA = 84 \]

3. Let \( R \) be the rectangle \([1, 3] \times [2, 8]\). Let

   \[ f(x, y) = \begin{cases} 
   -2 & \text{if } y < 4 \\
   5 & \text{if } y \geq 4 
   \end{cases} \]

   \[ \iint_R f(x, y) \, dy \, dx = -8 + 40 = 32 \]