Math 233 - October 27, 2009

- If a vector field satisfies the mixed partial condition, $\operatorname{curl} F = (0, 0, 0)$, how can we tell if F is conservative on its entire domain?
- How do we integrate in $\mathbb{R}^2$?

1. Let

$$F = \left(\frac{y^2 - 3y + x^2}{y^2 + x^2}, -\frac{y^2 + x^2 - 3x}{y^2 + x^2} \right)$$

(a) With $F = (f, g)$, find f_y and g_x. Is $f_y = g_x$?

(b) Let $r(t) = (\cos t, \sin t)$, $0 \leq t \leq 2\pi$. Compute

$$\frac{1}{2\pi} \oint_r F \cdot dr =$$

(c) Find, if possible, k and ϕ so that $F = kG + \nabla\phi$ where $G = (-y/(x^2 + y^2), x/(x^2 + y^2))$.

$$k =$$

$$\nabla\phi =$$

$$\phi =$$

1. Let

$$F = \left(\frac{y^2 - 3y + x^2}{y^2 + x^2}, -\frac{y^2 + x^2 - 3x}{y^2 + x^2} \right)$$

(a) With $F = (f, g)$, find f_y and g_x. Is $f_y = g_x$?
Solution:

$$f_y = g_x = \frac{3(y - x)(y + x)}{(y^2 + x^2)^2}$$

(b) Let $r(t) = (\cos t, \sin t)$, $0 \leq t \leq 2\pi$. Compute

$$\frac{1}{2\pi} \oint_r F \cdot dr = 3$$

(c) Find, if possible, k and ϕ so that $F = kG + \nabla\phi$ where $G = (-y/(x^2 + y^2), x/(x^2 + y^2))$.

$$\begin{aligned} k &= 3 \\ \nabla\phi &= (1, -1) \\ \phi &= x - y \end{aligned}$$

Lecture Problems

2. Let R be the rectangle $[1,3] \times [2,8]$. Let

$$f(x,y) = 7$$

$$\iint_R f(x,y)\, dA =$$

3. Let R be the rectangle $[1,3] \times [2,8]$. Let

$$f(x,y) = \begin{cases} -2 & \text{if } y < 4 \\ 5 & \text{if } y \geq 4 \end{cases}$$

$$\iint_R f(x,y)\, dy\, dx =$$

Lecture Problems

2. Let R be the rectangle $[1,3] \times [2,8]$. Let

$$f(x,y) = 7$$

$$\iint_R f(x,y)\, dA = 84$$

3. Let R be the rectangle $[1,3] \times [2,8]$. Let

$$f(x,y) = \begin{cases} -2 & \text{if } y < 4 \\ 5 & \text{if } y \geq 4 \end{cases}$$

$$\iint_R f(x,y)\, dy\, dx = -8 + 40 = 32$$